Problem Set 2

Instructor: Prof. Yun S. Song Due: February 7, 2013, in the beginning of class.

1. Recall that n events $A_1, \ldots, A_n$ are said to be independent if

$$\mathbb{P}(B_1 \cap \cdots \cap B_n) = \prod_{i=1}^{n} \mathbb{P}(B_i), \qquad \text{for all } B_i \in \{A_i, A_i^c\}, i = 1, \ldots, n. \tag{1}$$

As discussed in class, another definition of independence is

$$\mathbb{P}(\cap_{i \in S} A_i) = \prod_{i \in S} \mathbb{P}(A_i), \qquad \text{for all subsets } S \subset \{1, \ldots, n\} \text{ with size } |S| \geq 2. \tag{2}$$

Note that (2) imposes $2^n - n - 1$ constraints on the probability distribution, while (1) defines 2^n constraints. It turns out that exactly $n + 1$ constraints implied by (1) are actually redundant. Explain this.

2. Do the following problems from the textbook: 1.6 (8); 2.1 (4, 6, 10); 2.2 (2, 6, 14)