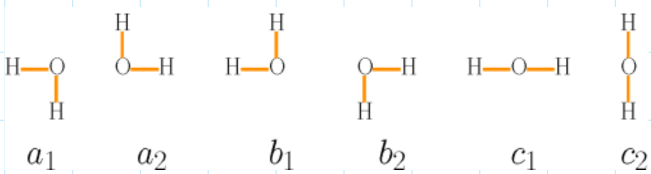
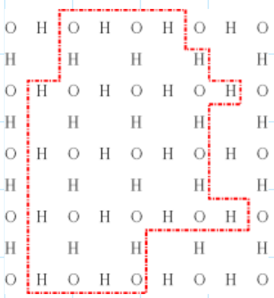
Topic: connection

Side 1 : Random configurations of (square ice) 6v model. in planar domains.

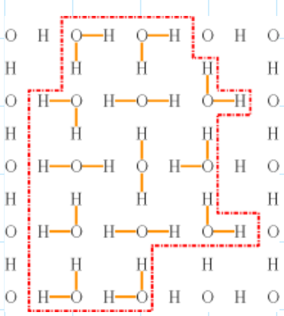


A) 6 molecules



B) Finite piece of H-O infinite grid

C) Matchings of H and O into molecules



D) Probability distribution

$$\mathrm{Prob}(\text{configuration}) \sim \prod_{\text{molecules}} \text{weight}(\text{molecule})$$

↑ $a_1, a_2, b_1, b_2, c_1, c_2$

[PDF] The structure and entropy of ice and of other crystals with some randomness of atomic arrangement
L Pauling - Journal of the American Chemical Society, 1935 - ACS Publications
… presumably lead to zero **entropy** for **any** system. **crystals** will be found to have residual **entropy** at very low temperatures as a result of **some randomness** of **atomic arrangement**. It is probable, however, that experimental verification of the residual **entropy** would be difficult for most of the cases mentioned below. …
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Residual entropy of **square ice**
EH Lieb - Physical Review, 1967 - APS
… At low temperatures, **ice** has a **residual entropy**, presumably … If we ignore the **ice** condition, then Z—=8'~=2'~ (because there are 2$ bonds) and W=4. This leads to an **entropy** that is …
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[BOOK] **Exactly solved** models in statistical mechanics
RJ **Baxter** - ~~2016~~ 1982 - books.google.com
This book was conceived as a slim monograph, but grew to its present size as I attempted to set down an account of two-dimensional lattice models in statistical mechanics, and how …
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[HTML] **Square ice** in graphene nanocapillaries
…, FC Wang, RR Nair, U Kaiser, HA Wu, AK **Geim**… - Nature, 2015 - nature.com
… The disagreement is perhaps not surprising when we consider that as P increases to reach the crystallization transition, hydrogen bonds switch to the in-plane configuration (Extended …
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Central mathematical question: For a very large domain how does a random configuration look like?

Side 2: Eigenvalues of self-adjoint random matrices

Side 2: Eigenvalues of self-adjoint random matrices

Setup: Take $N \times N$ self-adjoint random matrix, specify in some way the law of its matrix elements

$$\begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & & \\ a_{N1} & \cdots & a_{NN} \end{pmatrix} \qquad a_{ij} = \overline{a_{ji}}$$

Look at eigenvalues $\lambda_1 \le \lambda_2 \le \dots \le \lambda_N$

Central question: If N is very large, how do eigenvalues look like?

[BOOK] **Random matrices**
ML Mehta - ~~2004~~ 1967 - books.google.com
Random Matrices gives a coherent and detailed description of analytical methods devised to study **random matrices**. These methods are critical to the understanding of various fields in …
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[BOOK] Spectral analysis of large dimensional random matrices
Z Bai, JW Silverstein - 2010 - Springer
The aim of this book is to investigate the spectral properties of random matrices (RM) when their dimensions tend to infinity. All classical limiting theorems in statistics are under the …
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[BOOK] An introduction to **random matrices**
GW Anderson, A Guionnet, O Zeitouni - 2010 - books.google.com
… This project started as notes for a class on **random matrices** that two of us (GA and O. Z.) taught in the University of Minnesota in the fall of 2003, and notes for a course in the probability …
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[BOOK] Eigenvalue distribution of large random matrices
LA Pastur, M Shcherbina - 2011 - books.google.com
Random matrix theory is a wide and growing field with a variety of concepts, results, and techniques and a vast range of applications in mathematics and the related sciences. The book, …
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[BOOK] **Log-gases** and random matrices (LMS-34)
PJ Forrester - 2010 - books.google.com
… account of these developments, emphasizing **log-gases** as a physical picture and heuristic, as … Peter **Forrester** presents an encyclopedic development of **log-gases** and random matrices …
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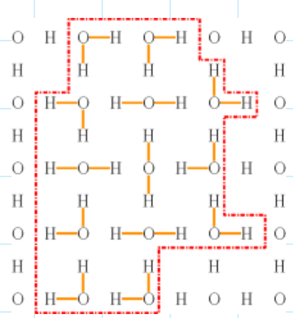
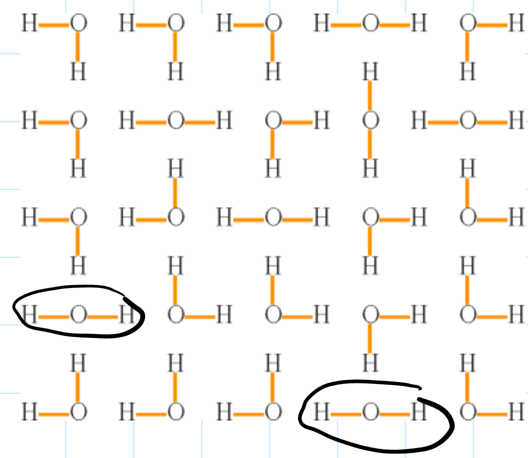


$$\longleftrightarrow \quad \begin{pmatrix} a_{11} & & \\ & \ddots & \\ & & a_{NN} \end{pmatrix}$$

Main message: these two topics are related

Example 1: Domain wall boundary condition

$N \times N$ square



H O H . . H on top

H
H
⋮
on the left

H
⋮
H on the right

H O . . H on the bottom

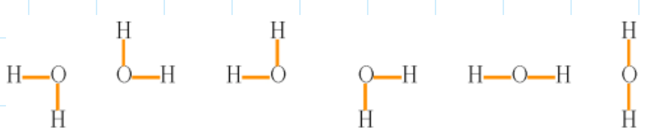
Exercise: In level k from the bottom there are m molecules H-O-H and $1 \le m \le k$

$k=1$:

Trace 1st row: starts from H-O (with H above) molecules on the left, ends by O-H (with H above) molecules on the right

single H-O-H molecule where one corner changes to another one

Theorem 1: (Johansson-Nordenstam-06) Suppose $a_1=a_2=b_1=b_2=1$, $c_1 = c_2 = 2$



a_1 a_2 b_1 b_2 c_1 c_2

Then as $N \to \infty$ there are exactly k H-O-H in row k with probability tending to 1, let $x_1^k < x_2^k \dots < x_k^k$ be their coordinates, then

$$\lim_{N\to\infty} \left\{ \frac{x_i^k - \gamma_1 N}{\gamma_2 \sqrt{N}} \right\}_{\dots} = \left\{ \lambda_i^k \right\}_{1 \le i \le k}$$

$$\lim_{N\to\infty} \left\{ \frac{\lambda_i - \gamma_1 N}{\gamma_2 \sqrt{N}} \right\}_{1\le i\le k} = \{\lambda_i\}_{1\le i\le k}$$

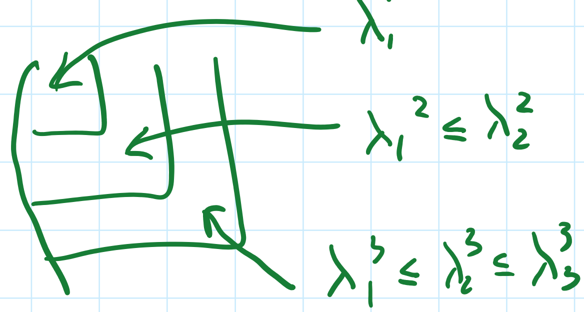
with $\gamma_1 = \frac{1}{2} = \gamma_2$ and $\{\lambda_i^k\}$ - „GUE-corners process"

$X = N\times N$ matrix of $N(0,1) + i\,N(0,1)$ independent matrix elements

$M = \frac{1}{2}(X + X^*)$ - „Gaussian Unitary Ensemble"

λ_i^k - i-th eigenvalue of principal $k\times k$ corner of M

$\lambda^1 = N(0,1)$



Theorem 2 (G.-14, G.-Panova-15) Same is true for $a_1 = a_2 = b_1 = b_2 = c_1 = c_2 = 1$ (uniform measure) with

$\gamma_1 = \frac{1}{2}$, $\gamma_2 = \sqrt{\frac{3}{8}}$ $\quad$ k was finite

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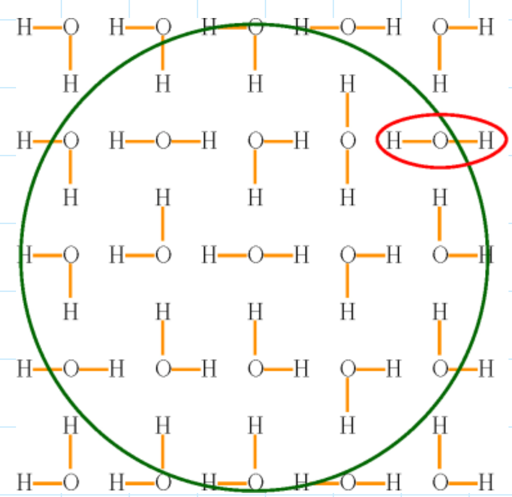
Th 3 (Johansson -00,05)

$a_1 = \dots = b_2 = 1 \quad c_1 \cdot c_2 = 2$

Draw an inscribed circle:



1) As $N\to\infty$ H-O-H stay inside the inscribed circle with prob tending to 1

2) For $k = \lfloor \alpha N \rfloor$ (integer part), $0<\alpha<1$, rightmost H-OH (position of it) satisfies

satisfies

[one-dim convergence] $\dfrac{x_k^k - \gamma_3(\alpha) N}{\gamma_4(\alpha) N^{1/3}} \xrightarrow{N\to\infty} TW_2$

position of it

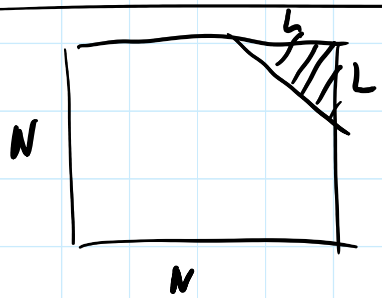
right edge of the circle

TW_2 = Tracy-Widom distribution at $\beta=2$

$$= \lim_{N\to\infty} \frac{\lambda_N^N - 2\sqrt{N}}{N^{-1/6}}$$

largest e.v. of GUE random matrix

„2" stays for the fact that we deal with complex matrices



ξ – maximal L such that there are no horizontal molecules in top-right $L\times L$ triangle

Theorem 4: (Johansson -05)

$$\lim_{N\to\infty} \frac{\xi - \gamma_5 N}{\gamma_6 N^{1/3}} = TW_1$$

$a_1=a_2=b_1=b_2=1 \quad c_1 c_2=2$

TW_1 is like TW_2, but dealing with real symmetric (instead of complex Hermitian) matrices.

Theorem 5] (Ayyer-Chhita-Johansson -22) Same is true for the uniform measure $(a_1=a_2=b_1=b_2=c_1=c_2=1)$ with different values of constants.

Conjecture: Theorems 1-5 extend (perhaps

Conjecture: Theorems 1-5 extend (perhaps with different values of constants) ~~for~~ to all $a_1, a_2, b_1, b_2, c_1, c_2$ satisfying

$$\Delta = \frac{a_1 a_2 + b_1 b_2 - c_1 c_2}{2\sqrt{a_1 a_2 b_1 b_2}} < 1$$

and much more general boundary conditions.
(domains)

$\Delta > 1$ will be discussed as well later

Next 4 lectures: Tue + Wednesday – fully self-contained proof of some cases of the above five theorems based on Izergin-Korepin determinant

Th + Fri: treatment of "stochastic six-vertex model" ($\Delta > 1$) by a "generalization" of IK-determinant.