

Stat 133, Spr '08
HW 4
Due Tue 18 March

Simulation of a Birth and Assassination Process¹

Consider the birth and assassination problem:

- The head of the family is born at time 0, and is assassinated at a random time X , which has an exponential distribution.
- While alive, the head has children according to a Poisson process with rate λ .
- As soon as they are born, the children produce offspring, also according to Poisson process with rate λ . And their children produce offspring according to the Poisson(λ) process, and so on.
- When the head is assassinated, then his children become vulnerable to assassination. The time between when the parent is assassinated and a child is assassinated follows the exponential distribution, and is independent of the siblings assassination times.
- The process continues in this manner, where the offspring are protected from assassination until their parent is assassinated. Then they are subject to assassination (at a random exponential time), and while alive they continue to produce offspring (and their offspring produce offspring, ...) at rate λ .

For this assignment, you are to write a function that will simulate this stochastic process.

The name of this function will be **BAgen**. The inputs will be λ the birth rate; κ the assassination rate; and *maxGen* the maximum number of generations (of course the family could die out before reaching this maximum).

¹This process was proposed and studied by Professor Aldous

The output will be a list with one element per generation. Each element will be a data frame, with have one row for each person born in that generation. The data frame will have four variables, *parentID* the identifier for the parent, *childID* the identifier for the child (a number from 1 to n , the total number of children born in that generation), birth date, assination date.

Some information that you may find useful about these distributions:

- A Poission(λ) process on a fixed interval, say $(0, t)$, will have a random number of hits in that interval, where the number of hits follows the Poisson(λt) distribution. A hit in this case is the birth of a child.
- The location of these hits follows the uniform distribution on $(0,t)$.

In addition to the function, you are to run the simulation for various values of λ and κ to see if you can determine which values will lead to the family dying out. Also, provide evidence that your simulation is functioning correctly. This evidence could be in the form of summary statistics, such as the average number of children each person has (over many runs of the simulation), the average life span of a person in a generation, etc. or in the form of plots.