Whose Votes (Were) Counted in the Election of 2016?

Philip B. Stark

Department of Statistics, University of California, Berkeley

24 January 2017
My connection to this election

- Op-ed with Ron Rivest calling for audit
- Testified against SD RoV for not following CA audit law
- Petition to audit the election (>330k signatures)
- Conversations with state & local election officials re auditing
- Conversation with Clinton campaign after election re audits & recounts
- Worked with Stein campaign on recount effort:
  - Testimony in Wisconsin
  - Affidavit in Michigan
- Report & op-eds regarding Maryland’s not-really-an-audit
- Dozens of interviews, radio & tv appearances, etc.
University of California
Professor Phillip Stark
367 Evans Hall
Berkeley, CA 94720

Dear Professor Stark,

Thank you for the entertaining gnashing of teeth since President Trump’s election win last November. This should be a lesson to your class and your colleagues that you cannot always get what you want.

Please repeat after me, P-R-E-S-I-D-E-N-T T-R-U-M-P!

Despite your loss, I have enclosed a 2016 presidential election participation award trophy to make you feel better about yourself. Feel free to share this participation trophy with the other whiners and snowflakes in your student body as well as the other professors. Thank you for the entertainment.

Sincerely,
Noneya Business

Figure 1: award
Yesterday’s news


Yesterday’s news


- How could you check?
Sniff test

- ~11 million aliens living in the US
Sniff test

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- Includes people of all ages
Sniff test

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- For the sake of argument, suppose that 75% are over the age of 18, i.e., 8.25 million.
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Turnout rate would need to be $5/8.25 = 61\%$ among illegal aliens for them to account for 5 million votes: higher than the overall turnout, which was about 56\%.
Statistical check

- Suppose 3 million people voted illegally, among the 139 million or so who voted: 2.16%.
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- Imagine drawing a random sample of 250 voters from whole US.
Statistical check

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- Imagine drawing a random sample of 250 voters from whole US.

- If indeed 2.16% or more voted illegally, the chance that the sample finds at least one of them is

\[
1 - \Pr\{\text{none in sample}\} \approx 1 - (0.9784)^{250} = 99.6\%
\]
Hacking

- Hackers compromised DNC, Clinton campaign (incl. Podesta’s email)
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- Why stop there?
# Accuracy

<table>
<thead>
<tr>
<th>State</th>
<th>Margin (%)</th>
<th>Margin (votes)</th>
<th>Electoral votes</th>
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<tbody>
<tr>
<td>Michigan</td>
<td>0.22%</td>
<td>10,704</td>
<td>16</td>
</tr>
<tr>
<td>New Hampshire</td>
<td>0.37%</td>
<td>2,736</td>
<td>4</td>
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<td>0.72%</td>
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- Margins in some states comparable to intrinsic accuracy of the counting technology even without hacking.
- DREs can be hacked without leaving a trace (Internet voting worse!)
- Unusually high undervote rates in some places.
Memes

- Impossible to hack US voting systems: too decentralized & heterogeneous
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- No evidence of hacking
Facts

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- Nobody *looked* for evidence of hacking: need to check the paper
Marinette County’s vote total changed by almost 300 because some voters were given the wrong pens to mark ballots, the Wisconsin Elections Commission said.
Overview

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- Those that check generally don’t check well
- No state that I’m aware of has good rules to take care of the paper
- Recount showed that laws & regs make it hard: states & candidates sued to stop recounts!
Some recount laws (e.g., MI) have perverse consequences, e.g.,

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Recounts clumsy & expensive compared to good audits

Need laws requiring 3Cs:
create paper, take care of paper, check results against paper
What do we want election audits to do?

- Provide reliable evidence that the electoral outcome is correct
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- If outcome is wrong, correct it before it’s official
How can an audit correct a wrong outcome?

- If there’s an adequately accurate audit trail, the audit could in principle count all the votes by hand: gold standard
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- Shouldn’t overturn outcome on statistics alone
Why not just count all votes by hand?

- Unnecessarily expensive and time-consuming.
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- Instead, count by machine, and check a random sample by hand.

- Keep checking until there’s convincing evidence that the outcome is right—or until all ballots have been examined and the right outcome is known.
Controlling the chance of error

- Since the sample is drawn at random, there’s a chance a wrong outcome will escape correction—but we can make that chance as small as we want. Statistics says how
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- Generally, have to check more to make chance smaller
Random Sampling

“Stirring” is key to reducing work

- Don’t have to climb into the bathtub to tell if it’s hot: can just stick your toe in—if the water is stirred well.
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- Don’t have to climb into the bathtub to tell if it’s hot: can just stick your toe in—if the water is stirred well.

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- Don’t have to drink a whole pot of soup to tell if it’s too salty: a teaspoon is enough—if the pot has been stirred. (Doesn’t matter whether the pot holds 1q or 50gal.)
How do you stir ballots?

Random sampling is stirring

- Imagine numbering the ballots

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- Imagine numbering the ballots
- Write the numbers on ping-pong balls; put in a lotto machine

Easier to stir balls than ballots. Even easier to generate (pseudo-)random numbers

Still amounts to putting ballots into a huge cement mixer to stir them, then taking a "teaspoon" of ballots.
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- Current procedures for protecting, tracking, and accounting for ballots are spotty. Should be top priority!
Paper rules—if it is right

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- Risk limit *assumes* outcome is wrong in the hardest-to-find way: Max chance outcome won’t be corrected.
Figure 2: Rivest & Stark
### Tools for Comparison Risk-Limiting Election Audits

To hide or show everything but the tools, [click this link.](#)

#### Initial sample size

**Contest information**
- **Ballots cast in all contests:** 7116
- **Smallest margin (votes):** 61
- **Diluted margin:** 0.86%

**Contest 1. Contest name:** Supervisor, 2nd District

**Winners:**
- 2

**Reported votes:**
- **Candidate 1 Name:** Juliana Inman
  - **Votes:** 1772
- **Candidate 2 Name:** Mark Luce
  - **Votes:** 2690
- **Candidate 3 Name:** Mark Van Gorder
  - **Votes:** 1833

- [Add candidate to contest 1](#)
- [Remove last candidate from contest 1](#)

**Audit parameters**

- **Risk limit:** 10%
- **Expected rates of differences (as decimal numbers):**
  - Overstatements:
    - 1-vote: 0.01
    - 2-vote: 0.001
  - Understatements:
    - 1-vote: 0.001
    - 2-vote: 0.0091

**Starting size**
- [Round up 1-vote differences.](#)
- [Round up 2-vote differences.](#)

![Contest inputs](image)

**Figure 3:** Contest inputs
Figure 4: Dice for PRNG seed
Figure 5: Manifest
Ballot-polling Audits and Comparison Audits

- Ballot polling audit: sample ballots until there is strong evidence that looking at all of them would show the same election outcome.
  Like an exit poll—but of ballots, not voters.

- Comparison audit:
  1. Commit to vote subtotals (or CVRs), e.g., precinct-level results
  2. Check that the subtotals add up exactly to contest results
  3. Check subtotals by hand until there is strong evidence the outcome is right
Tradeoffs

- Ballot polling audit
  - Virtually no set-up costs
  - Requires nothing of voting system
  - Need a ballot manifest to draw sample
  - Preserves voter anonymity except possibly for sampled ballots
  - Requires more counting than ballot-level comparison audit
  - Does not check tabulation: outcome could be right because errors cancel

- Comparison audit
  - Heavy demands on voting system for reporting and data export
  - Requires LEO to commit to subtotals
  - Requires ability to retrieve ballots that correspond to CVRs or subtotals
  - May compromise voter privacy
  - Most efficient (ballot-level) not possible with current systems
  - Checks tabulation
  - Ballot-level comparison audits require least hand counting
Ballot-polling Audits are often Cheap for Big Contests

255 state-level presidential contests, 1992–2012, 10% risk limit

BPA expected to examine fewer than 308 ballots for half the contests.

Work expands as margins shrink, but we could get a lot of election integrity at low cost—with any paper-based system.
<table>
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<th>median</th>
<th>90th percentile</th>
<th>Mean</th>
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<tr>
<td>70%</td>
<td>22</td>
<td>60</td>
<td>30</td>
</tr>
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<td>38</td>
<td>108</td>
<td>53</td>
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<td>84</td>
<td>244</td>
<td>119</td>
</tr>
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<td>58%</td>
<td>131</td>
<td>381</td>
<td>184</td>
</tr>
<tr>
<td>55%</td>
<td>332</td>
<td>974</td>
<td>469</td>
</tr>
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<td>54%</td>
<td>518</td>
<td>1,520</td>
<td>730</td>
</tr>
<tr>
<td>53%</td>
<td>914</td>
<td>2,700</td>
<td>1,294</td>
</tr>
<tr>
<td>52%</td>
<td>2,051</td>
<td>6,053</td>
<td>2,900</td>
</tr>
<tr>
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<td>8,157</td>
<td>24,149</td>
<td>11,556</td>
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<td>32,547</td>
<td>96,411</td>
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Risk-Limiting Audits

- ~25 pilot audits in CA, CO, and OH; AZ *tomorrow*
- CO law goes into effect this year; CA has pilot law
- simple measures, super-majority, multi-candidate, vote-for-n
- multiple contests audited simultaneously with one sample
- contest sizes: 200 ballots to 121,000 ballots
- counting burden: 16 ballots to 7,000 ballots
- cost per audited ballot: nil to about $0.55
- several jurisdictions have audited on their own—no geeks needed
Evidence-based elections

Principle: Trust, but verify

- LEOs should give convincing evidence that outcomes are right (or say they can’t). “Trust me” is not convincing.
- Voters create complete, durable, accurate audit trail.
- LEO curates the audit trail adequately.
- Compliance audit to check whether the audit trail is trustworthy enough to determine who won. If not, how strong can the evidence be?
- Risk-limiting audit to correct the outcome if it is wrong. Presumes audit trail is OK.
Reading
Scholarly-ish articles

Popular media, letters to politicians, etc.:

- [https://epic.org/policy/SHSGAC_EPIC_Bossert_Jan_2017.pdf](https://epic.org/policy/SHSGAC_EPIC_Bossert_Jan_2017.pdf)