Risk-limiting Audits and Evidence-based Elections

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What do we want election audits to do?

Ensure that the electoral outcome is correct; If outcome is wrong, correct it before it's official.

How can an audit correct a wrong outcome?

If there's an adequately accurate audit trail, the audit could in principle count all the votes by hand.

(Never overturn outcome on statistics alone.)

Why not just count all votes by hand?

- Unnecessarily expensive and time-consuming.
- Instead, count by machine, and check a random sample by hand.
- Keep checking until there's convincing evidence that the outcome is right—or until all ballots have been examined and the right outcome is known.

Controlling the chance of error

- Since the sample is drawn at random, there's a chance a wrong outcome will escape correction—but we can make that chance as small as we want. Statistics says how.
- *Risk* is the largest possible chance that the audit does not correct the outcome, if the outcome is wrong.
- *Risk-limiting audit* ensures that the largest possible chance is still a small chance, like 5% or 1%.

Random Sampling

"Stirring" is key to reducing work

Don't have to drink a whole pot of soup to tell if it's too salty: a teaspoon is enough—if the pot has been stirred. (Doesn't matter whether the pot holds 1q or 50g.)

How do you stir ballots?

Random sampling is stirring

- Imagine numbering the ballots.
- Write the numbers on ping-pong balls; put in a lotto machine.
- Lotto machine stirs the balls thoroughly and spits some out.
- The ballots with the numbers on the selected balls are a random sample of ballots.
- In practice, use computers "seeded" with true randomness, e.g., from dice rolls.

Paper rules—if it is right

- Can't correct wrong outcomes without counting the whole audit trail.
- Counting the whole audit trail won't give true result unless it's adequately accurate and intact.
- Current procedures for protecting, tracking, and accounting for ballots are spotty. Should be top priority!

Ballot-polling Audits and Comparison Audits

 Ballot polling audit: sample ballots until there is strong evidence that looking at all of them would show the same election outcome.

Like an exit poll—but of ballots, not voters.

- Comparison audit:
 - 1. Commit to vote subtotals (or CVRs), e.g., precinct-level results
 - 2. Check that the subtotals add up exactly to contest results
 - 3. Check subtotals by hand until there is strong evidence the outcome is right

Tradeoffs

- Ballot polling audit
 - Virtually no set-up costs
 - Requires nothing of voting system
 - Need a ballot manifest to draw sample
 - Preserves voter anonymity except possibly for sampled ballots
 - Requires more counting than ballot-level comparison audit
 - Does not check tabulation: outcome could be right because errors cancel
- Comparison audit
 - Heavy demands on voting system for reporting and data export
 - Requires LEO to commit to subtotals
 - Requires ability to retrieve ballots that correspond to CVRs or subtotals
 - May compromise voter privacy
 - Most efficient (ballot-level) not possible w/ current systems: requires rescan
 - Checks tabulation (but not for *transitive audits* unless subtotals are cross checked as well)
 - Ballot-level comparison audits require least hand counting

Ballot-polling Audits are often Cheap for Big Contests

255 state-level presidential contests, 1992–2011, 10% risk limit

BPA expected to examine fewer than 308 ballots for half the contests.

Work expands as margins shrink, but we could get a lot of election integrity at low cost—with any paper-based system.

Workload estimate: Ballot-Polling Audit, 2 Candidates, 10% Risk Limit

Winner's	Ballots drawn			
True Share	median	90th percentile		
70%	22	60		
65%	38	108		
60%	84	244		
58%	131	381		
55%	332	974		
54%	518	1,520		
53%	914	2,700		
52%	2,051	6,053		
51%	8,157	24,149		
50.5%	32,547	96,411		

Workload estimate: 2016 U.S. Presidential Election 5% Risk Limit

About 700,000 ballots nationwide.

Workload estimate: ballot-level comparison audit 5% Risk Limit

About 6.3/margin. Margin 10%: 63 ballots. Margin 1%: 630 ballots. Margin 0.1%: 6300 ballots.

Much cheaper than a recount.

Risk-Limiting Audits

- >20 pilot audits in AZ, CA, CO, OH
- CO law requires RLAs starting this year; CA AB44, SB360 anticipate
- simple measures, super-majority, multi-candidate, vote-for-n
- multiple contests audited simultaneously with one sample
- contest sizes: 200 ballots to 121,000 ballots
- counting burden: 16 ballots to 7,000 ballots
- cost per audited ballot: nil to about \$0.55
- several jurisdictions have audited on their own—geeks optional

Evidence-based elections

Principle: Trust, but Verify

LEOs should give convincing evidence that outcomes are right (or say they can't).

"Trust me" is not convincing.

- Voters create complete, durable, accurate audit trail.
- LEO curates the audit trail.
- Compliance audit to check whether the audit trail is trustworthy enough to determine who won.
 If not, how strong can the evidence be?
- Risk-limiting audit to correct the outcome if it is wrong. Presumes audit trail is trustworthy.
 "Explaining" or "resolving" errors isn't enough.

RLA legislation for CA currently being drafted.

Role of certification

- Under laboratory conditions, can the vote tabulation system—as delivered from the manufacturer—count votes with a specified level of accuracy?
- 2. As maintained, deployed, and used in the current election, did the vote tabulation system find the true winners?

Certification can cost millions and take years. Addresses Q 1.

Q 2 seems more important. Audits address Q 2.

If a jurisdiction uses a certified system, costs more to use it as a component of a resilient canvass framework because auditing will be more expensive.

Moreover, audit is less transparent.

Certification still useful for some things, e.g., to ensure accessibility and creation of durable audit trail.

2012 Napa County, CA, Audit Tools for Comparison Risk-Limiting Election Audits

To hide or show everything but the tools, click this link.

Initial sample size

		3). 01. Di	neumary			
Contest 1. Contest name: Supervisor, 2nd District						
Winners: 2 ÷						
Reported votes:						
Candidate 1 Name: Juliana Iaman	Votes:	1770				
Candidate 2 Name: Mark Luce	Votes:	2606				
Candidate 3 Name: Mark Van Corder	Votes:	1833				
Add contest Remove last contest						
Add contest Remove last contest Audit parameters					 	
Add contest Remove last contest Audit parameters Risk limit: 10%				 	 	
Add contest Remove last contest Audit parameters Risk limit: 10% Expected rates of differences (as decimal number	rs):			 	 	
Add contest Remove last contest Audit parameters Risk limit: 10% Expected rates of differences (as decimal number Overstatements. 1-vote: 0.001 2-vote: 0.000	rs):			 	 	
Add contest Remove last contest Audit parameters	rs): 001			 	 	



Random sampling

–Pseudo-Random	Sample	e of	Ballots
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Seed: 73567556725160627585

Number of ballots: 7116

Current sample number: 623

Draw this many ballots: 623

draw sample reset

Ballots selected: Show sequence numbers is show hash values

sequence_number, ballot	
1,2086	
2,2462	
3,3320	
4,4719	
5,4813	
6,3838	
7,2655	
8,2747	
9,3059	

Ballots selected, sorted:

 $19,34,37,38,51,90,96,96,99,101,109,114,150,156,163,175,187,187,195,197,198,244,280,281,301,316,\\372,395,403,404,407,417,429,444,450,451,471,477,480,481,482,491,514,542,545,550,554,577,585,58\\5,596,597,613,614,615,629,645,647,657,685,692,694,739,750,763,768,792,795,798,819,832,841,\\842,857,862,871,874,876,884,901,906,923,923,934,937,937,958,963,973,978,1018,1049,1050,1071,1\\081,1097,1105,1125,1126,1130,1165,1205,1210,1218,1219,1224,1226,1284,1288,1291,1318,1327,13\\57,1370,1372,1388,1406,1422,1425,1432,1433,1434,1446,1447,1457,1484,1494,1496,1507,1512,152\\3,1524,1540,1572,1574,1575,1576,1611,1614,1626,1634,1638,1642,1644,1665,1677,1685,1718,1735,\\1761,1764,1774,1788,1791,1793,1816,1827,1851,1855,1893,1921,1978,1989,2010,2017,2034,2056,2\\056,2058,2062,2069,2083,2086,2100,2112,2152,2189,2192,2206,2208,2210,2213,2224,2249,2266,22\\91,2295,2302,2331,2332,2390,2391,2395,2398,2401,2422,2436,2462,2463,2474,2495,2513,2514,252\\$

Ballots selected, sorted, duplicates removed:

19,34,37,38,51,90,96,99,101,109,114,150,156,163,175,187,195,197,198,244,280,281,301,316,372,395,403,404,407,417,429,444,450,451,471,477,480,481,482,491,514,542,545,550,554,577,585,596,597,60,13,614,615,629,645,647,657,685,692,694,739,750,763,768,792,795,798,819,832,841,842,857,862,871,874,876,884,901,906,923,934,937,958,963,973,978,1018,1049,1050,1071,1081,1097,1105,1125,1126,1130,1165,1205,1210,1218,1219,1224,1226,1284,1288,1291,1318,1327,1357,1370,1372,1388,1406,1422,1425,1432,1433,1434,1446,1447,1457,1484,1494,1496,1507,1512,1523,1524,1540,1572,1574,1575,1576,1611,1614,1626,1634,1638,1642,1644,1665,1677,1685,1718,1735,1761,1764,1774,1788,1791,1793,1816,1827,1851,1855,1893,1921,1978,1989,2010,2017,2034,2056,2058,2062,2069,2083,2086,2100,2112,2152,2189,2192,2206,2208,2210,2213,2224,2249,2266,2291,2295,2302,2331,2332,2390,2391,2395,2398,2401,2422,2436,2462,2463,2474,2495,2513,2514,2520,2549,2556,2558,2563,2578,255

Find ballots using a ballot manifest

Ballot look-up tool

Ballot manifest: Each line must have a batch label, a comma, and one of the following:

(i) the number of ballots in the batch

(ii) a range specified with a colon (e.g., 131:302), or

(iii) a list of ballot identifiers within parentheses, separated by spaces (e.g., (996 998 1000)). Each line should have exactly one comma.

001_211161_01,23	
002_211162_02,9	
003_211561_03,32	
004_211561_03,50	
005_211561_03,50	
006_211562_04,14	
007_211562_04,50	
008_211562_04,50	
009_211562_04,50	
010_211563_05,12	
011_211751_06,27	
012_211761_07,2	
013_211761_07,50	
014_211761_07,50	
015_211761_07,50	
016_211761_07,50	
017_211771_08,2	
018_221161_09,16	
019_221161_09,50	
020_221161_09,50	
021_221161_09,50	
022_221162_10,30	
023_221162_10,50	
024_221162_10,50	
025_221162_11,50	

Ballots to look up (separated by commas):

 $19,34,37,38,51,90,96,96,99,101,109,114,150,156,163,175,187,187,195,197,198,244,280,281,301,316,\\372,395,403,404,407,417,429,444,450,451,471,477,480,481,482,491,514,542,545,550,554,577,585,58\\5,596,597,613,614,615,629,645,647,657,685,692,692,694,739,750,763,768,792,795,798,819,832,841,\\842,857,862,871,874,876,884,901,906,923,923,934,937,937,958,963,973,978,1018,1049,1050,1071,1\\081,1097,1105,1125,1126,1130,1165,1205,1210,1218,1219,1224,1226,1284,1288,1291,1318,1327,13\\57,1370,1372,1388,1406,1422,1425,1432,1433,1434,1446,1447,1457,1484,1494,1496,1507,1512,152\\3,1524,1540,1572,1574,1575,1576,1611,1614,1626,1634,1638,1642,1644,1665,1677,1685,1718,1735,\\1761,1764,1774,1788,1791,1793,1816,1827,1851,1855,1893,1921,1978,1989,2010,2017,2034,2056,2\\056,2058,2062,2069,2083,2086,2100,2112,2152,2189,2192,2206,2208,2210,2213,2224,2249,2266,22\\91,2295,2302,2331,2332,2390,2391,2395,2398,2401,2422,2436,2462,2463,2474,2495,2513,2514,252\\$



look up ballots Sorted look up table:
Sorted lookup table: sorted_number, ballot, batch_label, which_ballot_in_batch 1, 19, 001_211161_01, 19 2, 34, 003_211561_03, 2 3, 37, 003_211561_03, 5 4, 38, 003_211561_03, 6 5, 51, 003_211561_03, 19 6, 90, 004_211561_03, 26 7, 96, 004_211561_03, 32
8, 96, 004_211561_03, 32 9, 99, 004_211561_03, 35 10, 101, 004_211561_03, 37 11, 109, 004_211561_03, 45 12, 114, 004_211561_03, 45 13, 150, 005_211561_03, 36 14, 156, 005_211561_03, 42
15, 163, 005_211561_03, 49 16, 175, 006_211562_04, 11 17, 187, 007_211562_04, 9 18, 187, 007_211562_04, 9 19, 195, 007_211562_04, 17 20, 197, 007_211562_04, 19 21, 198, 007_211562_04, 20 22, 244_009_211562_04, 26
22, 244, 008_211562_04, 16 23, 280, 009_211562_04, 2 24, 281, 009_211562_04, 3

Should more ballots be audited?

Ballots audited so far: 623		
1-vote overstatements: 0	Rate: 0	
2-vote overstatements: 0	Rate: 0	
1-vote understatements: 0	Rate: 0	
2-vote understatements: 0	Rate: 0	
Estimated stopping size		
Calculate Audit complete		

2008 Yolo County, CA Measure W Audit





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programs; architecture, engineering and construction management; computer scient early childhood development; and fire and police public safety programs at the Americ River; Cosumnes River; El Dorado, Folson and Sacramento City College campuses?"



2009 Yolo County, CA Measure P Audit

Special Election November 2009			
City of Davis November 03, 2009	Precir	Special Election November 2009 City of Davis November 03, 2009	Precinc
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2011 Orange County, first audit under AB 2023





















Project	wrongly mark, tear or deface any portion poll worker. If you are a vote-by-mail vote Voters office at 1300-C South Grand Aver (714) 567-7600, to obtain another belief.
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at 8:00 p.m. Postmarks are not ac

NONPARTISAN

CITY OF SAN CLEMENTE

A-San Clemente, Playa del Norte Co Project

Shall Resolution No. 10-53 approving commercial development project in the adopted?

Yes No

Contest: Vote for 1



commercial develo adopted? These Second No	10000250100033	at 8:00 p.m. Postmarks are n NONPARTISAN CITY OF SAN CLEMENTE A-San Clemente, Playa del Nor Project Shall Resolution No. 10-53 appr commercial development project adopted?
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CITY OF SAN CLEMENTE

A-San Clemente, Playa del Project

Shall Resolution No. 10-53 commercial development pri adopted?





commercial develo adopted?





Contest: Vote for 1

1000060100022

NONPARTISAN

CITY OF SAN CLEM A-San Clemente, Play Project Shall Resolution No. commercial developm adopted?





NONPARTISAN CITY OF SAN CLE A-San Clemente, Pl Project Shall Resolution No commercial develop adopted? Yes

No

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