Through the looking-glass, or sabbatical adventures in post-2020 election integrity

Department of Statistics
University of California
Berkeley, CA

Philip B. Stark
31 August 2022

University of California, Berkeley
Many collaborators including (most recently) Andrew Appel, Josh Benaloh, Michelle Blom, Andrew Conway, Rich DeMillo, Amanda Glazer, Alex Halderman, Harri Hursti, Wojciech Jamroga, Mark Lindeman, Kellie Ottoboni, Aaditya Ramdas, Ron Rivest, Peter Røenne, Peter Ryan, Steve Schneider, Carsten Schürmann, Jake Spertus, Peter Stuckey, Vanessa Teague, Poorvi Vora, Damjan Vukcevic, Dan Wallach, Ian Waudby-Smith
In Torrent of Falsehoods, Trump Claims Election Is Being Stolen

Most television networks cut away from the statement President Trump gave Thursday night from the White House briefing room on the grounds that what he was saying was not true.

“If you count the legal votes, I easily win,” President Trump said Thursday night in an unusually subdued, 17-minute televised statement from the lectern in the White House briefing room. Doug Mills/The New York Times
(Reuters) - About half of all Republicans believe President Donald Trump “rightfully won” the U.S. election but that it was stolen from him by widespread voter fraud that favored Democratic President-elect Joe Biden, according to a new Reuters/Ipsos opinion poll.

The Nov. 13-17 opinion poll showed that Trump’s open defiance of Biden’s victory in both the popular vote and Electoral College appears to be affecting the public’s confidence in American democracy, especially among Republicans.
JOINT STATEMENT FROM ELECTIONS INFRASTRUCTURE GOVERNMENT COORDINATING COUNCIL & THE ELECTION INFRASTRUCTURE SECTOR COORDINATING EXECUTIVE COMMITTEES

Original release date: November 12, 2020

WASHINGTON – The members of Election Infrastructure Government Coordinating Council (GCC) Executive Committee – Cybersecurity and Infrastructure Security Agency (CISA) Assistant Director Bob Kolasky, U.S. Election Assistance Commission Chair Benjamin Hovland, National Association of Secretaries of State (NASS) President Maggie Toulouse Oliver, National Association of State Election Directors (NASED) President Lori Augino, and Escambia County (Florida) Supervisor of Elections David Stafford – and the members of the Election Infrastructure Sector Coordinating Council (SCC) – Chair Brian Hancock (Unisyn Voting Solutions), Vice Chair Sam Derheimer (Hart InterCivic), Chris Wlaschin (Election Systems & Software), Ericka Haas (Electronic Registration Information Center), and Maria Bianchi (Democracy Works) - released the following statement:

"The November 3rd election was the most secure in American history. Right now, across the country, election officials are reviewing and double checking the entire election process prior to finalizing the result.

"When states have close elections, many will recount ballots. All of the states with close results in the 2020 presidential race have paper records of each vote, allowing the ability to go back and count each ballot if necessary. This is an added benefit for security and resilience. This process allows for the identification and correction of any mistakes or errors. There is no evidence that any voting system deleted or lost votes, changed votes, or was in any way compromised.

"Other security measures like pre-election testing, state certification of voting equipment, and the U.S. Election Assistance Commission's (EAC) certification of voting equipment help to build additional confidence in the voting systems used in 2020.

"While we know there are many unfounded claims and opportunities for misinformation about the process of our elections, we can assure you we have the utmost confidence in the security and integrity of our elections, and you should too. When you have questions, turn to elections officials as trusted voices as they administer elections."
Trump supporters file lawsuit asking Georgia to decertify election, declare Trump the winner

Sidney Powell files voting lawsuit in Ga.
Sidney Powell shares 270-page binder of documents buttressing election fraud claims

by Daniel Chaitin, Breaking News Editor | 📰 | December 27, 2020 08:56 PM
| Updated Dec 27, 2020, 10:33 PM
Lin Wood Doxed Georgia Officials to Hundreds of Thousands of QAnon Supporters

The pro-Trump lawyer asked an ‘Army of Patriots’ on Telegram to dig up dirt on officials who will decide whether he is disbarred or not.

February 15, 2021, 3:58am  F  Share  T  Tweet  A  Print

Pro-Trump lawyer and major QAnon booster Lin Wood has urged hundreds of thousands of his supporters to dig up dirt on Georgia officials who will decide if he should be disbarred or not — and to help them in their research, Wood published the officials’ addresses on social media.

In a 1,600-page filing, the State Disciplinary Board of the State Bar of Georgia said that it had “received information concerning the above-named attorney that suggests that said attorney may have violated one or more of the Georgia Rules of Professional Conduct.”
Pro-Trump Lawyer Lin Wood Is Investigated for Alleged Illegal Voting in Georgia

Wood promoted claims that the election was rigged against former President Donald Trump

Lin Wood said he decided on Monday that he would change his residency to South Carolina, and that he has lived in Georgia since 1995.
PHOTO: BEN MARGOT/ASSOCIATED PRESS

By Alexa Corse and Erin Allworth
Feb 2, 2021 11:43 pm ET
Sidney Powell’s secret ‘military intelligence expert,’ key to fraud claims in election lawsuits, never worked in military intelligence
Republicans Are P-Hacking the Supreme Court

Texas is seeking to overturn the 2020 election based on a shoddy statistical analysis. It's just what you would expect from medical researchers.

I spent the last month watching, with alternating apprehension and delight, as President Trump's cynical legal efforts to overturn the presidential election deteriorated into absurdity. After dozens of lawsuits were thrown out of court, and votes were certified in contested states, I thought we'd reached the end of the road. But it turns out there was one gut punch left to deliver, a bright red line no science-minded person like myself can bear to see crossed. That's right, Donald Trump misused statistics.

The Texas attorney general filed a lawsuit Monday asking the US Supreme Court to intervene in the election. Before your heart rhythm changes too dramatically, I should tell you that legal experts consider the case "doomed." That doesn't mean the lawsuit can't be dangerous. It introduced the strange-but-real number "quadrillion" into the political discourse for a couple of news cycles and seeded a new set of numerical curiosity theories that could live on for years as so-called proof of election fraud. On Tuesday, as more states prepared to back the Texas lawsuit, press secretary Kayleigh McEnany tweeted out one of its central claims: "Chances of Biden winning Pennsylvania, Michigan, Georgia, Wisconsin independently alter @realDonaldTrump's early lead is less than one in a quadrillion." She then proceeded to type out the number with all of its 15 glorious zeroes.
WASHINGTON – The Supreme Court on Monday formally rejected a handful of cases related to the 2020 election, including disputes from Pennsylvania that had divided the justices just before the election.

The cases the justices rejected involved election challenges filed by former President Donald Trump and his allies in five states President Joe Biden won: Arizona, Georgia, Michigan, Pennsylvania and Wisconsin.

Other than the disputes from Pennsylvania, the justices' decision not to hear the cases was unsurprising. The court had previously taken no action in those cases and in January had turned away pleas that the cases be fast-tracked, again suggesting the justices were not interested in hearing them.
Sidney Powell Drops Georgia Suit, Marking End to Presidential Election-Related Lawsuits in State

BY NICOLE FELLERT ON 1/19/21 AT 5:00 PM EST

Politicians And Celebrities React To Georgia Senate Election Results As Democrats Take Control Of Senate
Politics
Dominion sues Giuliani over false election fraud claims
January 26, 2021 | 12:51 PM PST

Voting machine company Dominion filed a $1.3 billion lawsuit against former president Donald Trump’s lawyer Rudy Giuliani on Jan. 25.

Related
Giuliani wasn’t just a Trump partisan but a shrewd marketer of vitamins, gold, lawsuit says
Dominion sues pro-Trump lawyer Sidney Powell, seeking more than $1.3 billion
Dominion Sues MyPillow, CEO Mike Lindell Over Election Claims

The voting-machine maker's lawsuit alleges defamation, seeks more than $1.3 billion in damages

Mike Lindell, CEO of Minnesota-based MyPillow, outside the West Wing of the White House on Jan. 15.

PHOTO: STAR TRIBUNE/ZUMA PRESS

By Alexa Corse
Updated Feb 22, 2021 9:02 am ET

WASHINGTON—One of the largest makers of voting machines in the U.S. on Monday sued a prominent supporter of former President Donald Trump, alleging that the businessman had defamed the company with false accusations that it had rigged the 2020 election for Joe Biden.
S. Ct. Case No. _________
11th Cir. Case No. 20-14418
N.D. Ga. Case No. 20-cv-04651-SDG

IN THE
SUPREME COURT OF THE UNITED STATES

L. LIN WOOD, JR.
Petitioner,

vs.

BRAD RAFFENSPERGER, et al.,
Respondents.

PETITION FOR WRIT OF CERTIORARI

On Petition for a Writ of Certiorari to the Eleventh Circuit Court of Appeals.

L. Lin Wood, Esq. (lead counsel)
GA Bar No. 774588
L. LIN WOOD, P.C.
P.O. BOX 82584
Atlanta, GA 30305-0584
(404) 891-1402
lwood@linwoodlaw.com

Harry W. MacDougald
Georgia Bar No. 463076
Caldwell, Probst & Deloach, LLP
FIFTH SUPPLEMENTAL DECLARATION OF PHILIP B. STARK

PHILIP B. STARK hereby declares as follows:

1. This statement supplements my declarations of September 9, 2018, September 30, 2018, October 22, 2019, and December 16, 2019. I stand by everything in the previous declarations.

2. Secretary of State Raffensperger issued the following (undated) press release on approximately June 30, 2020:1

   AUDIT SUPPORTS PRIMARY OUTCOME

   (ATLANTA) – A pilot post-election audit Monday confirmed the outcomes of the presidential preference primaries in Fulton County, Secretary of State Brad Raffensperger announced today.

   "This procedure demonstrates once again the validity of the results produced by Georgia’s new secure paper-ballot system," [SOS Raffensperger] said. "Auditing

Is Trump right about Georgia vote?

By Robert Sanders, Media relations | NOVEMBER 13, 2020
Why Georgia’s Unscientific Recount ‘Horrified’ Experts

Observers, including the inventor of the auditing process used by the state, were skeptical of a measure seemingly aimed at placating the GOP.

By Timothy Pratt

November 20, 2020
63. Fulton County did not produce the image file corresponding to every cast vote record. For the first machine count, production included images of ballots or BMD printout cards for only 168,726 of the 528,776 cast vote records: 376,863 image files are missing. For the second machine count, Fulton County’s production included images of ballots or BMD printout cards for 510,073 of the 527,925 cast vote records: 17,852 image files are missing.

64. Entire batches of images are missing from Fulton County’s production, for example, images from Scanner 801 batch 117 and Scanner 801 batch 118 are referred to in the cast vote records for the second machine count but the images were not among the electronic records. Without additional discovery it is impossible to determine whether the missing images are missing because of human error, programming errors (bugs), or malware in Fulton County’s election management system (EMS). Of course, those possibilities are not mutually exclusive.
47. The following table shows the counts of election-day votes in Fulton County precinct RW01 for the three presidential candidates, according to the original machine count, the machine recount, and the “audit,” and vote-by-mail and advance votes for the original election and the recount. (The audit did not report precinct-level results for vote-by-mail or advance voting.)

<table>
<thead>
<tr>
<th>Count</th>
<th>Election Day</th>
<th>Advance</th>
<th>Absentee by Mail</th>
<th>Provisional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trump</td>
<td>Biden</td>
<td>Jorgensen</td>
<td>Trump</td>
</tr>
<tr>
<td>Original</td>
<td>193</td>
<td>88</td>
<td>11</td>
<td>1455</td>
</tr>
<tr>
<td>Recount</td>
<td>162</td>
<td>73</td>
<td>9</td>
<td>1487</td>
</tr>
<tr>
<td>Audit</td>
<td>243</td>
<td>88</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Scanner</td>
<td>Batch</td>
<td>Mode of voting</td>
<td>Trump</td>
<td>Biden</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
<td>----------------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>absentee</td>
<td>4</td>
<td>93</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>absentee</td>
<td>6</td>
<td>92</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>12–14</td>
<td>?</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>239</td>
<td>?</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>80–84</td>
<td>?</td>
<td>118</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>absentee</td>
<td>30</td>
<td>66</td>
</tr>
<tr>
<td>7</td>
<td>AP01A-1</td>
<td>election day</td>
<td>84</td>
<td>62</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>179–181</td>
<td>absentee</td>
<td>85</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>239</td>
<td>absentee</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>Chastain</td>
<td>12</td>
<td>advance</td>
<td>613</td>
</tr>
<tr>
<td>11</td>
<td>Chastain</td>
<td>114</td>
<td>advance</td>
<td>613</td>
</tr>
</tbody>
</table>

40. I searched the audit spreadsheet for tallies that matched the numbers in these missing ABBSs. There are no data in the audit spreadsheet matching rows 4–11 of the table.

There are data that match rows 1, 2, and 3, but with distinctively different batch identifiers. It is plausible that these are genuinely different batches, and I have no reason to believe otherwise: some identical counts in different batches are to be
58, supra, Coalition Plaintiffs identified 12 hand-marked ballots from Fulton County precinct RW01 that were scanned twice in the first machine count (the original election). The pairs of images are listed in the table below. The format of the numbers is

\[
\text{[scanner number]} \_ \text{[batch number]} \_ \text{[image number]}.\]

<table>
<thead>
<tr>
<th>pair</th>
<th>Image A</th>
<th>Image B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>05162_00234_000096</td>
<td>05162_00235_000057</td>
</tr>
<tr>
<td>2</td>
<td>05162_00234_000093</td>
<td>05162_00235_000054</td>
</tr>
<tr>
<td>3</td>
<td>05162_00234_000074</td>
<td>05162_00235_000036</td>
</tr>
<tr>
<td>4</td>
<td>05162_00234_000072</td>
<td>05162_00235_000034</td>
</tr>
<tr>
<td>5</td>
<td>05162_00234_000068</td>
<td>05162_00235_000030</td>
</tr>
<tr>
<td>6</td>
<td>05162_00234_000069</td>
<td>05162_00235_000031</td>
</tr>
<tr>
<td>7</td>
<td>05162_00234_000054</td>
<td>05162_00235_000014</td>
</tr>
<tr>
<td>8</td>
<td>05162_00234_000031</td>
<td>05162_00235_000090</td>
</tr>
<tr>
<td>9</td>
<td>05162_00234_000026</td>
<td>05162_00235_000085</td>
</tr>
<tr>
<td>10</td>
<td>05162_00234_000017</td>
<td>05162_00235_000076</td>
</tr>
<tr>
<td>11</td>
<td>05162_00234_000013</td>
<td>05162_00235_000072</td>
</tr>
<tr>
<td>12</td>
<td>05162_00234_000014</td>
<td>05162_00235_000073</td>
</tr>
<tr>
<td>13</td>
<td>05162_00234_000003</td>
<td>05162_00235_000062</td>
</tr>
<tr>
<td>14</td>
<td>05162_00234_000001</td>
<td>05162_00235_000060</td>
</tr>
</tbody>
</table>
73. Coalition Plaintiffs also identified one hand-marked paper ballot that was scanned twice in RW01 in the machine recount, and at least seven hand-marked paper ballots that were scanned thrice in RW01 in the machine recount. I used the software in Appendix 6 to check their work: the twenty-nine images indeed seem to represent only eleven distinct pieces of paper, even though they contributed twenty-nine votes to some contests, including the presidential contest. Appendix 8 shows the sets of images. The table below lists the pairs and triples.

<table>
<thead>
<tr>
<th>Multiple</th>
<th>Image A</th>
<th>Image B</th>
<th>Image C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>00801_00044_000168</td>
<td>00801_00043_000168</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>00801_00044_000083</td>
<td>00801_00043_000083</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>00801_00044_000042</td>
<td>00801_00043_000042</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>05160_00074_000023</td>
<td>05160_00067_000008</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>00794_00017_000024</td>
<td>00791_00026_000091</td>
<td>00791_00019_000010</td>
</tr>
<tr>
<td>6</td>
<td>00794_00017_000029</td>
<td>00791_00026_000086</td>
<td>00791_00019_000015</td>
</tr>
<tr>
<td>7</td>
<td>00794_00018_000001</td>
<td>00791_00026_000009</td>
<td>00791_00019_000092</td>
</tr>
<tr>
<td>8</td>
<td>00794_00018_000011</td>
<td>00791_00026_000019</td>
<td>00791_00019_000082</td>
</tr>
<tr>
<td>9</td>
<td>00794_00019_000002</td>
<td>00791_00026_000079</td>
<td>00791_00019_000022</td>
</tr>
<tr>
<td>10</td>
<td>00794_00019_000005</td>
<td>00791_00026_000076</td>
<td>00791_00019_000025</td>
</tr>
<tr>
<td>11</td>
<td>00794_00019_000006</td>
<td>00791_00026_000075</td>
<td>00791_00019_000026</td>
</tr>
</tbody>
</table>
ATLANTA (AP) — The prosecutor investigating whether former President Donald Trump and others illegally tried to interfere in the 2020 election in Georgia is seeking information about the alleged involvement of a Trump ally in the breach of voting equipment at a county roughly 100 miles south of her Atlanta office.
<table>
<thead>
<tr>
<th></th>
<th>Recount</th>
<th>Griffin, r</th>
<th>Recount</th>
<th>Lynn, r</th>
<th>Recount</th>
<th>McMahon, r</th>
<th>Recount</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4,480</strong></td>
<td>4,777</td>
<td>5,292</td>
<td>5,591</td>
<td>4,786</td>
<td>5,089</td>
<td>5,256</td>
<td>5,554</td>
</tr>
<tr>
<td><strong>297</strong></td>
<td>299</td>
<td></td>
<td>303</td>
<td></td>
<td></td>
<td></td>
<td>298</td>
</tr>
<tr>
<td>St. Laurent, d</td>
<td>Recount</td>
<td>Azibert, d</td>
<td>Recount</td>
<td>Roman, d</td>
<td>Recount</td>
<td>Singureanu, d</td>
<td>Recount</td>
</tr>
<tr>
<td><strong>4,456</strong></td>
<td>4,357</td>
<td>2,787</td>
<td>2,808</td>
<td>3,415</td>
<td>3,443</td>
<td>2,764</td>
<td>2782</td>
</tr>
</tbody>
</table>

33,236  34,401
STATE OF NEW HAMPSHIRE

In the Year of Our Lord Two Thousand Twenty One

AN ACT authorizing an audit of the Rockingham County district 7 state representative race.

Be it Enacted by the Senate and House of Representatives in General Court convened:

2:1 Purpose. Notwithstanding any provisions of law to the contrary, this act authorizes and directs an audit of the ballot counting machines and their memory cards and the hand tabulations of ballots regarding the general election on November 3, 2020 in Windham, New Hampshire of Rockingham County district 7 house of representatives for the purpose of determining the accuracy of the ballot counting
SB 43 Forensic Audit

Forensic Election Audit Team's Report
* Forensic Election Audit Teams Report
* Ballot Inspection Instructions
* Tally Instructions
* Tally Sheets

Joint Report of the Secretary of State and Attorney General Regarding the Senate Bill 43 Audit

Windham Forensic Audit Right-to-Know Requests & Responses

Forensic Audit Team
* Mark Linden
* Harri Hunt
* Philip Stark

Windham Audit Misc. Files
* Live Data Entry to Hand Tally
* Machine Count Post Tapes from May 14, 2021
* Windham Sample Ballot
* Example Unmarked Filled Windham Ballot
* 2019 Oregon Ballot Text to Tapes
* Ball Counting Weekly Postcards for May 14, 2021
* Election Material Container Manifest
* Windham Ballot Chain of Custody
* 2016 General Election Area from Windham to the State Archives in November 2020

Daily Audit Documents
- Wednesday, May 19, 2021
- Tuesday, May 18, 2021
- Monday, May 17, 2021
- Friday, May 14, 2021
- Thursday, May 13, 2021
- Wednesday, May 12, 2021
- Tuesday, May 11, 2021
- Monday, May 10, 2021
- Friday, May 7, 2021
- Thursday, May 6, 2021
- Wednesday, May 5, 2021
- Tuesday, May 4, 2021
- Monday, May 3, 2021
- Friday, May 1, 2021
- Thursday, April 30, 2021
- Wednesday, April 28, 2021
- Tuesday, April 27, 2021
- Monday, April 26, 2021
- Friday, April 23, 2021
- Thursday, April 22, 2021
- Wednesday, April 21, 2021
- Tuesday, April 20, 2021
- Monday, April 19, 2021
- Friday, April 16, 2021
- Thursday, April 15, 2021
- Wednesday, April 14, 2021
- Tuesday, April 13, 2021
- Monday, April 12, 2021
Audit was conducted in a stand-alone building located inside a military compound. The building was surrounded by an easy-to-monitor perimeter.

2) Location, premises, chain of custody, and security transparency

   a) The audit was conducted in a Regional Training Institute building located inside the New Hampshire Army National Guard Pembroke Readiness Center.
   b) Additional security support on the base gate and outside of the building was provided by New Hampshire State Police. The building perimeter was protected in person 24/7 by state troopers.
   c) Inside the building, additional security was provided by New Hampshire Department of Justice investigators.
   d) Building access control and monitoring systems were provided by the New Hampshire Army National Guard and programmed to permit access to the building by a limited number of access cards. All use of the access cards was logged by the National Guard.
   e) Security-sensitive materials, such as spare seals, were removed from the room each night and given to the State Trooper on duty for overnight safekeeping. The State Trooper on duty did not have an access card to enter the building, but had a complete view into the working room through large windows.
   f) A livestream of the room, with audio, was broadcast 24/7.\(^*\) Multiple physical clocks were positioned strategically in the room and visible in the livestreams.
   g) Security seals of voting equipment were publicly inspected multiple times during the audit; the inspections were livestreamed.
   h) Material under chain-of-custody was placed closest to the windows, allowing those outside of the room a clear view. In particular, the State Trooper on duty
A panoramic view of the room taken from the far corner of the observation area during the hand recount phase of the audit. Screens on the perimeter tables were providing a close-up view of the documents on the hand count tables.

Cameras were realigned to provide views from all angles, including over-the-shoulder in high detail and zoomed in as needed during the machine hardware audit.
Ballot flow in the room during the machine recount process. To facilitate observability, the ballot movements shown in blue arrows and green arrows alternated and never happened at the same time.

9) Locating all cast ballots:
   a) Volunteers opened each election material container and determined its contents: cast ballots, other election materials, or a combination.
   b) Cast ballots immediately proceeded to the next step.
   c) Other election materials were reboxed for further scrutiny at a later step. (One box contained a combination of cast ballots and other election materials, so a new box was created for the other election materials at this step.)

10) Creating scan batches and adding ballot IDs:
    a) Volunteers divided the ballots in each box into one or more “scan batches.” Each scan batch had an orange scan batch cover sheet. (Most scan batches contained between 150 and 300 ballots.)
    b) On the bottom of the back of each ballot, volunteers wrote a unique ballot ID number in red ink. Bates stamps designated for this purpose broke down, so most ballot IDs were hand-written by the volunteers.
6) Workflow, key design parameters

a) Only one type of activity took place in the working area at any given time, prioritizing clarity and observability over speed or efficiency.
   i) This allowed the four official video cameras to capture all the activities in the room.
   ii) It increased the predictability of processes and made it easier to have all materials under control during the lunch break and at the end of each day.
   iii) It allowed the activities to be organized in a way that kept the most important action as close as possible to the observers.

b) We managed inventory and flow control over the materials subject to chain of custody to make those materials as easy as possible for the audience to track visually.
   i) Only a limited number of ballots were at any given time unsealed out of the boxes.
   ii) Only a limited number of ballots were on the processing tables at any time.
   iii) Only workers who were clearly identifiable (wearing bright yellow vests) were allowed to move ballots from one station to another.

c) When activities changed, there was a clear demarcation.
   i) The current activity was finalized.
   ii) The transition was announced.
   iii) The floor plan was set up for the next activity.
   iv) Details of the next workflow were (re)announced.
   v) Instructions were given to the volunteers as needed.
   vi) The next workflow began.
- everything livestreamed; observers allowed, outside “wire”
- data products pushed to web daily
- inventory all materials, log chain of custody
- write identifiers on all ballots (red/green ink)
- high-resolution images of all ballots
- tabulate all ballots on all 4 machines
- image memory cards and EPROMS
- forensic examination of ballot paper
- fiber optic inspection of scanners for dust
- only sworn election officials touched ballots/machines until retabulation done
<table>
<thead>
<tr>
<th>candidate</th>
<th>Hand count</th>
<th>Machine count*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2020</td>
<td>2021</td>
</tr>
<tr>
<td></td>
<td>diff</td>
<td></td>
</tr>
<tr>
<td>St. Laurent</td>
<td>4357</td>
<td>4355</td>
</tr>
<tr>
<td>Azibert</td>
<td>2808</td>
<td>2808</td>
</tr>
<tr>
<td>Roman</td>
<td>3443</td>
<td>3442</td>
</tr>
<tr>
<td>Singareanu</td>
<td>2782</td>
<td>2782</td>
</tr>
<tr>
<td>Soti</td>
<td>4777</td>
<td>4776</td>
</tr>
<tr>
<td>Griffin</td>
<td>5591</td>
<td>5591</td>
</tr>
<tr>
<td>Lynn</td>
<td>5089</td>
<td>5089</td>
</tr>
<tr>
<td>McMahon</td>
<td>5554</td>
<td>5554</td>
</tr>
<tr>
<td>write in</td>
<td>n/a</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>Machine 1</td>
<td>Machine 2</td>
</tr>
<tr>
<td>--------------------------</td>
<td>-----------</td>
<td>-----------</td>
</tr>
<tr>
<td>overvote/undervote rate in State Rep contest machine count, 2020</td>
<td>14.5%</td>
<td>19.3%</td>
</tr>
</tbody>
</table>
Isn’t your time too valuable to spend stuffing envelopes? Now the DS35 can do it for you, simply and efficiently. The Neopost DS35 Folder Inserter neatly prepares your documents for mailing with ease and accuracy. It’s quiet. It’s simple. And you’ll wonder how you ever got along without it.

- State-of-the-art full color touch screen
- 15 programmable jobs
- 2 sheet and 1 insert/BRE feeders
- No. 10 up to 6 x 9.5 envelopes
- secure’n feed double detection
<table>
<thead>
<tr>
<th>Machine</th>
<th>head first face up</th>
<th>head first face down</th>
<th>foot first face up</th>
<th>foot first face down</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 (78355)</td>
<td>27</td>
<td>46</td>
<td>46</td>
<td>27</td>
</tr>
<tr>
<td>“school” (81365)</td>
<td>54</td>
<td>54</td>
<td>52</td>
<td>41</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Machine</th>
<th>head first face up</th>
<th>head first face down</th>
<th>foot first face up</th>
<th>foot first face down</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 (78355)</td>
<td>25</td>
<td>24</td>
<td>27</td>
<td>15</td>
</tr>
<tr>
<td>“school” (81365)</td>
<td>43</td>
<td>32</td>
<td>37</td>
<td>24</td>
</tr>
</tbody>
</table>

import numpy as np
from PIL import Image, ImageDraw, ImageFont
import os, sys, re, itertools
import cv2
import time
from cryptorandom import cryptorandom as cr
from cryptorandom import cryptorandom as cr
from cryptorandom import cryptorandom as cr
from permute import utils import hypergeom_conf_interval

# Contest
CONTEST = 'gov'  # which contest to look at: 'rep' for State Representative, 'gov' for Governor

# fold detection settings
KERNEL_WIDTH = 5  # width of Gaussian kernel for blurring the area with fold lines
CANNY_LO = 50  # Thresholds for Canny filter before Hough
CANNY_HI = 150
RHO = 1  # distance resolution in pixels of the Hough grid
THETA_RES = np.pi / 400  # angular resolution in radians of the Hough grid
HOUGH_THRESH = 60  # minimum number of votes (intersections in Hough grid cell)
MIN_HOUGH_LINE_LEN = 100  # minimum number of pixels making up a line (consider raising to 100)
MAX_HOUGH_LINE_GAP = 25  # maximum gap in pixels between connectable line segments
LINE_DETECT_METHOD = 'fld'  # method for detecting lines. Use 'Hough' for probabilistic Hough method, 'fld' for fast line detector
CANNY_LENGTH_THRESHOLD = 50  # for fld
CANNY_APERATURE_SIZE = 5  # for fld
SLOPE_LIMIT = 0.8  # ignore lines with slopes larger than this

# image registration settings
MAX_FEATURES = 5000  # for warping ballot images to the template
GOOD_MATCH_FRAC = 0.15  # quantile of best features to keep for warping ballot images
SIDE_MATCH_THRESH = 0.6  # threshold for matching to front or back

# vote detection settings
VOTE_THRESH = 200  # mean gray value threshold in a vote target to consider it a marked oval

# reporting settings
SKIP = 0  # number of image files to skip, for parallel processing
## Absentee Ballot and Official Ballot for Windham General Election November 3, 2020

### Instructions to Voters

1. To Vote: Complete all the votes to the right of your choice. For each office, vote for one candidate only. If your ballot is completed in such a manner that your vote for a given office is not completed, your vote for that office will not be counted.

2. To Vote by Mail: To vote by mail, complete a vote by mail ballot as directed on the back of this ballot. Place the completed ballot in the sealed envelope and mail it in the upper right-hand corner of the ballot. Be sure to affix the prepaid postcard with your mailing address and known date of arrival. Your ballot must be postmarked by the Thursday before the election and received by the Windham Town Clerk’s Office by 4:00 p.m. on the day of the election to be counted.

### Offices

- **President and Vice-President of the United States**
  - Write-In

- **Governor**
  - Darryl W. Perry
  - Dan Felle
  - Chris Sununu

- **United States Senator**
  - Justin O’Donnell
  - Joanne Shaheen
  - Corly Messner

- **Representative in Congress**
  - Andrew J. O’Malley
  - Ann McLane Kuster
  - Steven Negron

- **Executive Councilor**
  - Richard S. Hiltz
  - Janet Stevens

- **State Senators**
  - Joshua Bourdon
  - John Berteau

- **State Representatives**
  - Krist St. Laurent
  - Jared W. Kost

### Candidates

- **President and Vice-President of the United States**
  - Write-In

- **Governor**
  - Darryl W. Perry
  - Dan Felle
  - Chris Sununu

- **United States Senator**
  - Justin O’Donnell
  - Joanne Shaheen
  - Corly Messner

- **Representative in Congress**
  - Andrew O’Malley
  - Ann McLane Kuster
  - Steven Negron

- **Executive Councilor**
  - Richard S. Hiltz
  - Janet Stevens

- **State Senators**
  - Joshua Bourdon
  - John Berteau

- **State Representatives**
  - Krist St. Laurent
  - Jared W. Kost

### BALLOT CONTINUES ON BACK - TURN OVER

---

~260

~4
Verified extra possibles: 268
Verified over possibles: 609
Ratio: 268/609 = 0.44

Expected handcount extras: ~22

Estimated lost for St. Laurent: 99+22 = 121
Estimated extras for Rs: 299-22 = 277
Ratio: 121/277 = 0.44
Other things we’ve checked:

Number of ballots: 10,006 +/- 2, as reported

Pollbook signatures

Absentee applications and envelopes (found three uncounted ballots)

Forensic inspection of ballots: printing, ink, hand-marking, paper thickness

Memory cards

EPROMs
Trump Cheered An Election Review In This Tiny New Hampshire Town — Auditors Just Said There’s No Proof Of Fraud

Joe Walsh Forbes Staff
I cover breaking news for Forbes.

May 27, 2021, 06:13pm EDT

Updated May 27, 2021, 06:13pm EDT

When New Hampshire began auditing a state legislative race in the town of Windham this month, former President Donald Trump suggested proof of widespread voter fraud could be lurking in the small suburban community — but Windham’s auditors on Thursday concluded there’s no evidence of vote-rigging in the town, in another disappointment for Trump supporters who have failed to substantiate any of the wild claims of nationwide fraud in the November election.
EMERGENCY ALERT

HACKING DEMOCRACY
NEW HAMPSHIRE

IRREFUTABLE EVIDENCE
OF VOTER FRAUD IN THE NH 2020 ELECTION

A KOOCHPIX PRODUCTION TITLED HACKING DEMOCRACY NEW HAMPSHIRE
It does not answer the question and you’re going to jail Harri!!! You never audited the card Harri!! You put it in supervisor mode twice Harri!!! And let’s also not forget that....
Kevin ➔ WindhamNHAuditors • 6 days ago

Now the crappy machines are breaking down, too? How much is Soro's paying you guys? I will never vote unless it is with paper ballots. I knew when Bush allowed machine ballots the sheep were in big trouble. Ken is a hero, and thankfully a Christian, we win in the end, Satan doesn't get much time.

3 ⬆ | ⬇ • Reply • Share

Jovan Hutton Pulitzer asks if 28% of ballots not counted in Windham, NH is fraud or failure

7 hr • 1 week ago

Editor’s note: The town of Windham, New Hampshire is presently conducting an audit of its Nov. 3, 2020 election. In which a hand recount one week later revealed that four Republican candidates each won by 99 votes not counted in Election Night. Vermont uses the same AccuVote-OV vote tabulators owned by Dominion and managed by LHS Associates, of Salem, N.H. The following article by Steve McDonald has been republished with permission from AccuVote.

By Steve McDonald

Jovan Hutton Pulitzer featured the Windham election and audit in a recent episode of his podcast. It’s not a lot of new information for our readers, but you’ll want to listen because what is new might amaze you.

He talks about balled ballots as part of the integrity of the process. A machine failure but a failure of people tasked with ensuring election integrity, missing or ignoring what appears to be the fraud in Windham, and possibly most of New Hampshire, going back decades.

The AccuVote machines can and will miss ballots that have been misfed – as in, mated – as in voted by mail. But not yet in 2002.

Alvin Luee discussed the problem in his guest the morning. The folks, the ballots, are not ballots, which mandate actual votes. In the case of Windham, the result is that 28% of ballots were likely not counted.

No company theories, no secret programming codes, an actual failure of the equipment to do what we pay it for, no the circumstances before or afterwards have been tested.

The problem, uncovered by the hand recount in Windham, is exactly what we’ve been trying to discover and what no one else wanted to address.

From the AP to the USA, of States, to the voting machine infrastructure

jack • 8 days ago

Assemble the gallows!

4 ⬆ | ⬇ • Reply • Share

GraniteGrok Mod ➔ jack • 7 days ago

This comment has been preserved and will not be deleted. Last night, I was informed that it has been turned into the NH Attorney General’s office as Philip Stark feels that he has been threatened.

I cannot remove it as it is now considered to be possible evidence.

A ⬆ | ⬇ • Reply • Share
“Congratulations to the great Patriots of Windham, New Hampshire for their incredible fight to seek out the truth on the massive Election Fraud which took place in New Hampshire and the 2020 Presidential Election,” Trump wrote on May 6 on his now-defunct blog. “People are watching in droves as these Patriots work tirelessly to reveal the real facts of the most tainted and corrupt Election in American history.”
In August 2021, some thoughtful person signed me up for:

- BeNaughty
- Blendr
- Christian Filipina
- Match
- OurTime
- QuickFlirt
- WantUBad
- Zoosk
IN THE UNITED STATES DISTRICT COURT
FOR THE DISTRICT OF COLUMBIA

PHILIP B. STARK and FREE SPEECH
FOR PEOPLE,

Plaintiffs,

v.

UNITED STATES ELECTION
ASSISTANCE COMMISSION,

Defendant.

Civil Action No. 1:21-cv-01864 (CKK)

PLAINTIFFS’ MEMORANDUM OF POINTS AND AUTHORITIES IN
OPPOSITION TO DEFENDANT’S MOTION TO DISMISS OR, IN THE
ALTERNATIVE, FOR PARTIAL SUMMARY JUDGMENT
A bit of research progress
RiLACS: Risk-Limiting Audits via Confidence Sequences

Ian Waudby-Smith\textsuperscript{1}, Philip B. Stark\textsuperscript{2}, and Aaditya Ramdas\textsuperscript{1}

\textsuperscript{1}Carnegie Mellon University
\textsuperscript{2}University of California, Berkeley

ianws@cmu.edu, stark@stat.berkeley.edu, aramdas@cmu.edu

November 16, 2021

Abstract

Accurately determining the outcome of an election is a complex task with many potential sources of error, ranging from software glitches in voting machines to procedural lapses to outright fraud. Risk-limiting audits (RLA) are statistically principled “incremental” hand counts that provide statistical assurance that reported outcomes accurately reflect the validly cast votes. We present a suite of tools for conducting RLAs using confidence sequences — sequences of confidence sets which uniformly capture an electoral parameter of interest from the start of an audit to the point of an exhaustive recount with high probability. Adopting the SHANGRLA [1] framework, we design nonnegative martingales which yield computationally and statistically efficient confidence sequences and RLAs for a wide variety of election types.
Non(c)esuch Ballot-Level Risk-Limiting Audits for Precinct-Count Voting Systems

Philip B. Stark

University of California, Berkeley, CA USA stark@stat.berkeley.edu

Abstract. Risk-limiting audits (RLAs) guarantee a high probability of correcting incorrect reported outcomes before the outcomes are certified. The most efficient RLAs use ballot-level comparison, comparing the voting system's interpretation of individual ballot cards sampled at random (cast-vote records, CVRs) from a trustworthy paper trail to a human interpretation of the same cards. Such comparisons require the voting system to create and export CVRs in a way that can be linked to the individual ballots the CVRs purport to represent. Such links can be created by keeping the ballots in the order in which they are scanned or by printing a unique serial number on each ballot. But for precinct-count systems (PCOS), these strategies may compromise vote anonymity: the order in which ballots are cast may identify the voters who cast them. Printing a unique pseudo-random number ("cryptographic nonce") on each ballot card after the voter last touches it could reduce such privacy risks. But what if the system does not in fact print a unique number on each ballot or does not accurately report the numbers it printed? This paper gives two ways to conduct an RLA so that even if the system does not print a genuine nonce on each ballot or misreports the nonces it used, the audit's risk limit is not compromised (however, the anonymity of votes might be compromised). One method allows untrusted technology to be used to imprint and to retrieve ballot cards. The method is adaptive: if the technology behaves properly, this protection does not increase the audit workload. But if the imprinting or retrieval system misbehaves, the sample size the RLA requires to confirm the reported results when the results are correct is generally larger than if the imprinting and retrieval were accurate. If the reported outcome is incorrect, the audit still limits the risk conservatively, whether the imprinting and retrieval technology misbehaved or not.
ALPHA: AUDIT THAT LEARNS FROM PREVIOUSLY HAND-AUDITED BALLOTS

BY PHILIP B. STARK

1Department of Statistics, University of California, Berkeley, stark@stat.berkeley.edu

A risk-limiting election audit (RLA) offers a statistical guarantee: if the reported electoral outcome is incorrect, the audit has at most a known maximum chance (the risk limit) of not correcting it before it becomes final. BRAVO (Linderman, Stark and Yates, 2012), based on Wald’s sequential probability ratio test for the Bernoulli parameter, is the simplest and most widely tried method for RLAs, but it has limitations. It cannot accommodate sampling without replacement or stratified sampling, which can improve efficiency and are sometimes required by law. It applies only to ballot-polling audits, which are less efficient than comparison audits. It applies to plurality, majority, super-majority, proportional representation, and instant-runoff voting (IRV, using RAIRE (Blom, Stuckey and Teague, 2018)), but not to other social choice functions for which there are RLA methods. And while BRAVO has the smallest expected sample size among sequentially valid ballot-polling-with-replacement methods when the reported vote shares are exactly correct, it can require arbitrarily large samples when the reported reported winner(s) really won but the reported vote shares are incorrect. ALPHA is a simple generalization of BRAVO that (i) works for sampling with and without replacement, with and without weights, with and without stratification, and for Bernoulli sampling; (ii) works not only for ballot polling but also for ballot-level comparison, batch polling, and batch-level comparison audits; (iii) works for all social choice functions covered by SHANGRLA (Stark, 2020), including approval voting, STAR-Voting, proportional representation schemes such as D’Hondt and Hamilton, IRV, Borda count, and all scoring rules; and (iv) in situations where both ALPHA and BRAVO apply, requires smaller samples than BRAVO when the reported vote shares are wrong but the outcome is correct—five orders of magnitude in some examples. ALPHA includes the family of betting martingale tests in RILACS (Waudby-Smith, Stark and Ramdas, 2021), with a different betting strategy parametrized as an estimator of the population mean and explicit flexibility to accommodate sampling weights and population bounds that change with each draw. A Python implementation is provided.
Sweeter than SUITE: Supermartingale Stratified Union-Intersection Tests of Elections

Jacob V. Spertus and Philip B. Stark*

University of California, Berkeley, Department of Statistics
jakespertus@berkeley.edu; pbstark@berkeley.edu

Abstract. Stratified sampling can be useful in risk-limiting audits (RLAs), for instance, to accommodate heterogeneous voting equipment or laws that mandate jurisdictions draw their audit samples independently. We combine the union-intersection tests in SUITE, the reduction of RLAs to testing whether the means of a collection of lists are all \( \leq 1/2 \) of SHANGRLA, and the nonnegative supermartingale (NNSM) tests in ALPHA to improve the efficiency and flexibility of stratified RLAs. A simple, non-adaptive strategy for combining stratumwise NNSMs decreases the measured risk in the 2018 pilot hybrid audit in Kalamazoo, Michigan, USA by more than an order of magnitude, from 0.037 for SUITE to 0.003 for our method. We give a simple, computationally inexpensive, adaptive rule for deciding which stratum to sample next that reduces audit workload by as much as 74% in examples. We also present NNSM-based tests that are computationally tractable even when there are many strata, illustrated with a simulated audit stratified across California’s 58 counties.
More Style, Less Work: Card-style Data Decrease Risk-limiting Audit Sample Sizes

AMANDA K. GLAZER, JACOB V. SPERTUS, and PHILIP B. STARK, Department of Statistics, University of California, Berkeley

U.S. elections rely heavily on computers such as voter registration databases, electronic pollbooks, voting machines, scanners, tabulators, and results reporting websites. These introduce digital threats to election outcomes. Risk-limiting audits (RLAs) mitigate threats to some of these systems by manually inspecting random samples of ballot cards. RLAs have a large chance of correcting wrong outcomes (by conducting a full manual tabulation of a trustworthy record of the votes), but can save labor when reported outcomes are correct. This efficiency is eroded when sampling cannot be targeted to ballot cards that contain the contest(s) under audit. If the sample is drawn from all cast cards, then RLA sample sizes scale like the reciprocal of the fraction of ballot cards that contain the contest(s) under audit. That fraction shrinks as the number of cards per ballot grows (i.e., when elections contain more contests) and as the fraction of ballots that contain the contest decreases (i.e., when a smaller percentage of voters are eligible to vote in the contest). States that conduct RLAs of contests on multi-card ballots or RLAs of small contests can dramatically reduce sample sizes by using information about which ballot cards contain which contests—by keeping track of card-style data (CSD). For instance, CSD reduce the expected number of draws needed to audit a single countywide contest on a 4-card ballot by 75%. Similarly, CSD reduce the expected number of draws by 95% or more for an audit of two contests with the same margin on a 4-card ballot if one contest is on every ballot and the other is on 10% of ballots. In realistic examples, the savings can be several orders of magnitude.

CCS Concepts: • **Applied computing** ➔ **Voting/election technologies**; **Computers in other domains**; **Computing in government**;

Additional Key Words and Phrases: Elections, audits, risk-limiting, ballots, card-style, sampling

ACM Reference format:
https://doi.org/10.1145/3457907

Style is a way to say who you are without having to speak. — Rachel Zoe

1 INTRODUCTION
The principle of evidence-based elections is that elections should provide convincing evidence that the reported winners really won [29]. Evidence-based elections require a trustworthy record of the votes. Generally, that
They may look and look, yet not see: BMDs cannot be tested adequately

Philip B. Stark, Ran Xie

Bugs, misconfiguration, and malware can cause ballot-marking devices (BMDs) to print incorrect votes. Several approaches to testing BMDs have been proposed. In logic and accuracy testing (LAT) and parallel or live testing, auditors input known test votes into the BMD and check the printout. Passive testing monitors the rate of "spoiled" BMD printout, on the theory that if BMDs malfunction, the rate will increase noticeably. We show that these approaches cannot reliably detect outcome-altering problems, because: (i) The number of possible interactions with BMDs is enormous, so testing interactions uniformly at random is hopeless. (ii) To probe the space of interactions intelligently requires an accurate model of voter behavior, but because the space of interactions is so large, building an accurate model requires observing a huge number of voters in every jurisdiction in every election—more voters than there are in most jurisdictions. (iii) Even with a perfect model of voter behavior, the number of tests needed exceeds the number of voters in most jurisdictions. (iv) An attacker can target interactions that are expensive to test, e.g., because they involve voting slowly; or interactions for which tampering is less likely to be noticed, e.g., because the voter uses the audio interface. (v) Whether BMDs misbehave or not, the distribution of spoiled ballots is unknown and varies by election and possibly by ballot style: historical data do not help much. Hence, there is no way to calibrate a threshold for passive testing, e.g., to guarantee at least a 95% chance of noticing that 5% of the votes were altered, with at most a 5% false alarm rate. (vi) Even if the distribution of spoiled ballots were known to be Poisson, the vast majority of jurisdictions do not have enough voters for passive testing to have a large chance of detecting problems but only a small chance of false alarms.
A Declaration of Software Independence

Wojciech Jamroga¹, Peter Y. A. Ryan¹(✉), Steve Schneider², Carsten Schürmann³, and Philip B. Stark⁴

¹ University of Luxembourg, Esch-sur-Alzette, Luxembourg
  peter.ryan@uni.lu
² University of Surrey, Guildford, England
³ IT University of Copenhagen, Copenhagen, Denmark
⁴ University of California, Berkeley, USA

Abstract. A voting system should not merely report the outcome: it should also provide sufficient evidence to convince reasonable observers that the reported outcome is correct. Many deployed systems, notably paperless DRE machines still in use in US elections, fail certainly the second, and quite possibly the first of these requirements. Rivest and Wack proposed the principle of software independence (SI) as a guiding principle and requirement for voting systems. In essence, a voting system is SI if its reliance on software is “tamper-evident”, that is, if there is a way to detect that material changes were made to the software without inspecting that software. This important notion has so far been formulated only informally.

Here, we provide more formal mathematical definitions of SI. This exposes some subtleties and gaps in the original definition, among them: what elements of a system must be trusted for an election or system to be SI, how to formalize “detection” of a change to an election outcome, the fact that SI is with respect to a set of detection mechanisms (which must be legal and practical), the need to limit false alarms, and how SI applies when the social choice function is not deterministic.
Who Was that Masked Voter? The Tally Won’t Tell!

Peter Y. A. Ryan\textsuperscript{1,2}\textsuperscript{a}, Peter B. Roenne\textsuperscript{1,2}\textsuperscript{b}, Dimitar Ostrev\textsuperscript{1,2}\textsuperscript{c},
Fatima-Ezzahra El Oech\textsuperscript{1,2}\textsuperscript{d}, Najmeh Sorouch\textsuperscript{1,2}\textsuperscript{e},
and Philip B. Stark\textsuperscript{1,2}\textsuperscript{f}

\textsuperscript{1} Interdisciplinary Centre for Security, Reliability, and Trust, SnT,
University of Luxembourg, Luxembourg City, Luxembourg
\{peter.ryan, peter.roenne, dimitar.ostrev, fatima.ezzahra.elorche, najmeh.sorouch\}@uni.lu
\textsuperscript{2} Department of Statistics, University of California, Berkeley, CA, USA
stark@stat.berkeley.edu
\textsuperscript{3} ENS, CNRS, PSL Research University, Paris, France
fatima.ezzahra.elorche@ens.fr

Abstract. We consider elections that publish anonymised voted ballots
or anonymised cast-vote records for transparency or verification pur-
poses, investigating the implications for privacy, coercion, and vote sell-
ing and exploring how partially masking the ballots can alleviate these
issues.

Risk Limiting Tallys (RLT), which reveal only a random sample of bal-
lots, were previously proposed to mitigate some coercion threats. Mask-
ing some ballots provides coerced voters with plausible deniability, while
risk-limiting techniques ensure that the required confidence level in the
election result is achieved. Risk-Limiting Verification (RLV) extended this
approach to masking a random subset of receipts or trackers.

Here we show how these ideas can be generalised and made more
flexible and effective by masking at a finer level of granularity: at the
level of the components of ballots. In particular, we consider elections
involving complex ballots, where RLT may be vulnerable to pattern-
based vote buying. We propose various measures of verifiability and
coercion-resistance and investigate how several sampling/masking strate-
gies perform against these measures. Using methods from coding theory,
we analyse signature attacks, bounding the number of voters who can be
coerced. We also define new quantitative measures for the level of
coaercion-resistance without plausible deniability and the level of vote-
buying-resistance without “free lunch” vote sellers.

These results and the different strategies for masking ballots are of
general interest for elections that publish ballots for auditing, verifica-
tion, or transparency purposes.
Assertion-Based Approaches to Auditing Complex Elections, with Application to Party-List Proportional Elections

Michelle Blom\textsuperscript{1}, Jurlind Budurushi\textsuperscript{2}, Ronald L. Rivest\textsuperscript{3},
Philip B. Stark\textsuperscript{4}, Peter J. Stuckey\textsuperscript{5}, Vanessa Teague\textsuperscript{6},
and Danijan Vukcevic\textsuperscript{7,8,9,10}

\textsuperscript{1} School of Computing and Information Systems, University of Melbourne,
Parkville, Australia
\texttt{michelle.blom@unimelb.edu.au}

\textsuperscript{2} Cloudical Deutschland GmbH, Berlin, Germany
\texttt{jurlind.budurushi@cloudical.io}

\textsuperscript{3} Computer Science and Artificial Intelligence Laboratory,
Massachusetts Institute of Technology, Cambridge, MA, USA

\textsuperscript{4} Department of Statistics, University of California, Berkeley, CA, USA

\textsuperscript{5} Department of Data Science and AI, Monash University, Clayton, Australia

\textsuperscript{6} Thinking Cybersecurity Pty. Ltd., Melbourne, Australia

\textsuperscript{7} School of Mathematics and Statistics, University of Melbourne, Parkville, Australia

\textsuperscript{8} Melbourne Integrative Genomics, University of Melbourne, Parkville, Australia
\texttt{danijan.vukcevic@unimelb.edu.au}

\textbf{Abstract.} Risk-limiting audits (RLAs), an ingredient in evidence-based elections, are increasingly common. They are a rigorous statistical means of ensuring that electoral results are correct, usually without having to perform an expensive full recount—at the cost of some controlled probability of error. A recently developed approach for conducting RLAs, SHANGRLA, provides a flexible framework that can encompass a wide variety of social choice functions and audit strategies. Its flexibility comes from reducing sufficient conditions for outcomes to be correct to canonical 'assertions' that have a simple mathematical form.

Assertions have been developed for auditing various social choice functions including plurality, multi-winner plurality, super-majority, Hamiltonian methods, and instant runoff voting. However, there is no systematic approach to building assertions. Here, we show that assertions with linear dependence on transformations of the votes can easily be transformed to canonical form for SHANGRLA. We illustrate the approach by constructing assertions for party-list elections such as Hamiltonian free list elections and elections using the D'Hondt method, expanding the set of social choice functions to which SHANGRLA applies directly.
A Unified Evaluation of Two-Candidate Ballot-Polling Election Auditing Methods

Zhuoqin Huang\textsuperscript{1}, Ronald L. Rivest\textsuperscript{2}[0000–0002–7105–3690], Philip B. Stark\textsuperscript{3}[0000–0002–5771–9604], Vanessa Teague\textsuperscript{4,5}[0000–0003–2648–2560], and Damjan Vukcevic\textsuperscript{6}[0000–0001–7790–9566]

\textsuperscript{1} School of Mathematics and Statistics, University of Melbourne, Parkville, Australia
\textsuperscript{2} Computer Science & Artificial Intelligence Laboratory, Massachusetts Institute of Technology, USA
\textsuperscript{3} Department of Statistics, University of California, Berkeley, USA
\textsuperscript{4} Thinking Cybersecurity Pty. Ltd.
\textsuperscript{5} College of Engineering and Computer Science, Australian National University
\textsuperscript{6} Melbourne Integrative Genomics, University of Melbourne, Parkville, Australia
\texttt{damjan.vukcevic@unimelb.edu.au}

Abstract. Counting votes is complex and error-prone. Several statistical methods have been developed to assess election accuracy by manually inspecting randomly selected physical ballots. Two ‘principled’ methods are risk-limiting audits (RLAs) and Bayesian audits (BAs). RLAs use frequentist statistical inference while BAs are based on Bayesian inference. Until recently, the two have been thought of as fundamentally different. We present results that unify and shed light upon ‘ballot-polling’ RLAs and BAs (which only require the ability to sample uniformly at random from all cast ballot cards) for two-candidate plurality contests, which are building blocks for auditing more complex social choice functions, including some preferential voting systems. We highlight the connections between the methods and explore their performance.

First, building on a previous demonstration of the mathematical equivalence of classical and Bayesian approaches, we show that BAs, suitably calibrated, are risk-limiting. Second, we compare the efficiency of the methods across a wide range of contest sizes and margins, focusing on the distribution of sample sizes required to attain a given risk limit. Third, we outline several ways to improve performance and show how the mathematical equivalence explains the improvements.

Keywords: Statistical audit · Risk-limiting · Bayesian
Auditing Ranked Voting Elections with Dirichlet-Tree Models: First Steps*

Floyd Everest\textsuperscript{1}[0000–0002–2726–6736], Michelle Blom\textsuperscript{2}[0000–0002–0459–9917], Philip B. Stark\textsuperscript{3}[0000–0002–3771–9604], Peter J. Stuckey\textsuperscript{4}[0000–0003–2186–0459], Vanessa Teague\textsuperscript{5}[0000–0003–2648–2565], and Damjan Vukcevic\textsuperscript{1,6}[0000–0001–7780–9586]

\textsuperscript{1} School of Mathematics and Statistics, University of Melbourne, Parkville, Australia
\textsuperscript{2} School of Computing and Information Systems, University of Melbourne, Parkville, Australia
\textsuperscript{3} Department of Statistics, University of California, Berkeley, CA, USA
\textsuperscript{4} Department of Data Science and AI, Monash University, Clayton, Australia
\textsuperscript{5} Thinking Cybersecurity Pty. Ltd., Melbourne, Australia
\textsuperscript{6} Melbourne Integrative Genomics, University of Melbourne, Parkville, Australia
damjan.vukcevic@unimelb.edu.au

\textbf{Abstract.} Ranked voting systems, such as instant-runoff voting (IRV) and single transferable vote (STV), are used in many places around the world. They are more complex than plurality and scoring rules, presenting a challenge for auditing their outcomes: there is no known risk-limiting audit (RLA) method for STV other than a full hand count. We present a new approach to auditing ranked systems that uses a statistical model, a Dirichlet-tree, that can cope with high-dimensional parameters in a computationally efficient manner. We demonstrate this approach with a ballot-polling Bayesian audit for IRV elections. Although the technique is not known to be risk-limiting, we suggest some strategies that might allow it to be calibrated to limit risk.
Assessing the accuracy of the Australian Senate count
Key steps for a rigorous and transparent audit

Michelle Blom
michelle.blom@unimelb.edu.au

Philip B. Stark
stark@stat.berkeley.edu

Peter J. Stuckey
peter.stuckey@monash.edu

Vanessa Teague
vanessa.teague@anu.edu.au

Damjan Vukcevic
damjan.vukcevic@unimelb.edu.au

June 23, 2022

1 Introduction

Australian paper-based elections counted in a polling place are carefully designed
to be both privacy-preserving and verifiable. Scrutineers can watch each voter put
a ballot paper in each box (one for the Senate and one for the House of Represen-
tatives), then watch the manual count of first preferences, or the two-candidate-
preferred count, after the polls close. The counting of postal votes can also be
scrutinised, although postal voting entails considerable trust in the postal service.

Senate votes are electronically counted after the ballot papers are scanned and the
scans transformed into digital preferences in a hybrid human-automated process.
Evidence-Based Elections
- US elections neither *tamper evident* nor *resilient*. 
- US elections neither *tamper evident* nor *resilient*.
- Need systems/procedures that can provide strong evidence that the reported winners really won.
- US elections neither *tamper evident* nor *resilient*.
- Need systems/procedures that can provide strong evidence that the reported winners really won.
- *Every* electronic system is vulnerable to bugs, configuration errors, & hacking.
Security properties of paper

- tangible/accountable
- tamper evident
- human readable
- large alteration/substitution attacks require physical access & many accomplices
Security properties of paper

- tangible/accountable
- tamper evident
- human readable
- large alteration/substitution attacks require physical access & many accomplices

Not all paper is trustworthy

A just-released study says over ninety percent of errors introduced by ballot marking devices go undetected.

SERIOUS DESIGN FLAW IN ESS EXPRESSVOTE TOUCHSCREEN: “PERMISSION TO CHEAT”

SEPTEMBER 14, 2018 BY ANDREW APPEL

Kansas, Delaware, and New Jersey are in the process of purchasing voting machines with a serious design flaw, and they should reconsider while there is still time.

Over the past 15 years, almost all the states have moved away from paperless touchscreen voting systems (DREs) to optical-scan paper ballots. They’ve done so because if a paperless touchscreen is hacked to give fraudulent results, there’s no way to know and no way to correct; but if an optical scanner were hacked to give fraudulent results, the fraud could be detected by a random audit of the paper ballots that the voters actually marked, and corrected by a recount of those paper ballots.
Ballot-Marking Devices Cannot Ensure the Will of the Voters

Andrew W. Appel, Richard A. DeMillo, and Philip B. Stark

Published Online: 17 Sep 2020  
https://doi.org/10.1089/eli.2019.0619

Abstract

The complexity of U.S. elections usually requires computers to count ballots—but computers can be hacked, so election integrity requires a voting system in which paper ballots can be recounted by hand. However, paper ballots provide no assurance unless they accurately record the votes as expressed by the voters.

Voters can express their intent by indelibly hand-marking ballots or using computers called ballot-marking devices (BMDs). Voters can make mistakes in expressing their intent in either technology, but only BMDs are also subject to hacking, bugs, and misconfiguration of the software that prints the marked ballots. Most voters do not review BMD-printed ballots, and those who do often fail to notice when the printed vote is not what they expressed on the touchscreen. Furthermore, there is no action a voter can take to demonstrate to election officials that a BMD altered their expressed votes, nor is there a corrective action that election officials can take if notified by voters—there is no way to deter, contain, or correct computer hacking in BMDs. These are the essential security flaws of BMDs.

Risk-limiting audits can ensure that the votes recorded on paper ballots are tabulated correctly, but no audit can ensure that the votes on paper are the ones expressed by the voter on a touchscreen. Elections conducted on current BMDs cannot be confirmed by audits. We identify two properties of voting systems, contestability and defensibility, necessary for audits to confirm election outcomes. No available BMD certified by the Election Assistance Commission is contestable or defensible.
- Hand-marked paper ballots are a record of what the voter did.
- Machine-marked paper ballots are a record of what the machine did.
- BMDs make voters responsible for catching & correcting machine errors/bugs/hacks.
- Experiments & polling-place observations show few voters check BMD printout; fewer notice errors.
Madison, Oconee, and Oglethorpe. The study, dated January 22, 2021, was not published; its existence was discovered through a Georgia Open Records Act request by *The Atlanta Journal Constitution*. Dr. Gilbert does not mention this study.

7. The results of the Haynes and Hood (2021) study are summarized in the table below. Less than 19 percent of voters looked at the BMD printout for 5 seconds or more.

<table>
<thead>
<tr>
<th>Duration of glance</th>
<th>Percentage of voters</th>
</tr>
</thead>
<tbody>
<tr>
<td>did not look at all</td>
<td>20.0 percent</td>
</tr>
<tr>
<td>less than one second</td>
<td>31.3 percent</td>
</tr>
<tr>
<td>one to five seconds</td>
<td>29.9 percent</td>
</tr>
<tr>
<td>five seconds or more</td>
<td>18.8 percent</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>County</th>
<th>Contests</th>
<th>Minimum estimated time required to read 4 words per contest (seconds)</th>
<th>included in Haynes &amp; Hood (2021)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrow</td>
<td>21</td>
<td>17–37</td>
<td>yes</td>
</tr>
<tr>
<td>Clarke</td>
<td>16</td>
<td>13–28</td>
<td>yes</td>
</tr>
<tr>
<td>Jackson</td>
<td>24</td>
<td>19–42</td>
<td>yes</td>
</tr>
<tr>
<td>Madison</td>
<td>23</td>
<td>18–40</td>
<td>yes</td>
</tr>
<tr>
<td>Oconee</td>
<td>27</td>
<td>22–47</td>
<td>yes</td>
</tr>
<tr>
<td>Oglethorpe</td>
<td>19</td>
<td>15–33</td>
<td>yes</td>
</tr>
<tr>
<td>Dougherty</td>
<td>19</td>
<td>15–33</td>
<td>no</td>
</tr>
<tr>
<td>Fulton</td>
<td>20</td>
<td>16–35</td>
<td>no</td>
</tr>
</tbody>
</table>
Did the reported winner really win?

- Procedure-based vs. evidence-based elections
  - sterile scalpel v. patient’s condition
Did the reported winner really win?

- Procedure-based vs. evidence-based elections
  - sterile scalpel v. patient’s condition
- Any way of counting votes can make mistakes
- Every electronic system is vulnerable to bugs, configuration errors, & hacking
Did the reported winner really win?

- Procedure-based vs. evidence-based elections
  - sterile scalpel v. patient’s condition
- Any way of counting votes can make mistakes
- Every electronic system is vulnerable to bugs, configuration errors, & hacking
- Did error/bugs/hacking cause the wrong candidate(s) to appear to win?
Evidence-Based Elections

P.B. Stark and D.A. Wagner

Abstract—We propose an alternative to current requirements for certifying voting equipment and conducting elections. We argue that elections should be structured to provide convincing affirmative evidence that the reported outcomes actually reflect how people voted. This can be accomplished with a combination of software-independent voting systems, compliance audits, and risk-limiting audits. Together, these yield a resilient canvas framework: a fault-tolerant approach to conducting elections that gives strong evidence that the reported outcome is correct or reports that the evidence is not convincing. We argue that, if evidence-based elections are adopted, certification and testing of voting equipment can be relaxed, saving money and time and reducing barriers to innovation in voting systems—and election integrity will benefit. We conclude that there should be more regulation of the evidence trail and less regulation of equipment, and that compliance audits and risk-limiting audits should be required.

Keywords: elections, software-independent voting system, risk-limiting audit, resilient canvas framework EDCCS SEC-INTE, APP-INTE, APP-OTHE.

I. INTRODUCTION

IDEALLY, what should an election do? Certainly, an election should find out who won, but we believe it also should produce convincing evidence that it found the real winner—or report that it cannot. This is not automatic; it requires thoughtful design of voting equipment, carefully planned and implemented voting and vote counting processes, and rigorous post-election auditing.

While approximately 75% of US voters currently vote on equipment that produces a voter-verifiable paper record of the vote, about 28% vote on paperless electronic voting machines that do not produce such a record [1]. Because paperless electronic voting machines rely upon complex software and hardware, and because there is no feasible way to ensure that the voting software is free of bugs or that the hardware is executing the proper software, there is no guarantee that electronic voting machines record the voter’s votes accurately. And, because paperless voting machines preserve only an electronic record of the vote that cannot be directly observed by voters, there is no way to produce convincing evidence that the electronic record accurately reflects the voters’ intent. Internet voting shares the shortcomings of paperless electronic voting machines, and has additional vulnerabilities. Numerous failures of electronic voting equipment have been documented. Paperless voting machines in Carteret County, North Carolina inevitably lost 4,400 votes; other machines in Mecklenburg, North Carolina recorded 3,955 more votes than the number of people who voted; in Bernalillo County, New Mexico, machines recorded 2,700 more votes than voters; in Mahoning County, Ohio, some machines reported a negative total vote count; and in Fairfax, Virginia, county officials found that for every hundred or so votes cast for one candidate, the electronic voting machines subtracted one vote for her [2].

In short, when elections are conducted on paperless voting equipment:
Voting system properties needed to justify public trust

- (Strong) Software Independence
- Contestability
- Defensibility
Voting system properties needed to justify public trust

- (Strong) Software Independence
- Contestability
- Defensibility

DREs, BMDs, online voting have none of these properties.
Risk-Limiting Audits (RLAs, Stark, 2008)

- If there’s a trustworthy paper record of votes, can check whether reported winner really won.
  - Can manually count
  - If you accept a controlled risk of not correcting a wrong reported outcome, can save effort
A *risk-limiting audit* has a known maximum chance of not correcting the reported outcome if it’s wrong & never changes correct outcomes.
A risk-limiting audit has a known maximum chance of not correcting the reported outcome if it’s wrong & never changes correct outcomes.

Risk limit: largest possible chance of not correcting a wrong reported outcome, no matter where or how errors/problems occurred.
A *risk-limiting audit* has a known maximum chance of not correcting the reported outcome if it’s wrong & never changes correct outcomes.

*Risk limit*: largest possible chance of *not* correcting a wrong reported outcome, no matter where or how errors/problems occurred.

Establishing whether paper trail is trustworthy involves other processes, generically, *compliance audits* along w/ thorough canvass, ballot accounting, pollbook/participation reconciliation, eligibility verification, demonstrably secure chain of custody, etc.
A *risk-limiting audit* has a known maximum chance of not correcting the reported outcome if it’s wrong & never changes correct outcomes.

*Risk limit*: largest possible chance of *not* correcting a wrong reported outcome, no matter where or how errors/problems occurred.

Establishing whether paper trail is trustworthy involves other processes, generically, *compliance audits* along w/ thorough canvass, ballot accounting, pollbook/participation reconciliation, eligibility verification, demonstrably secure chain of custody, etc.

DRE & BMD printout is not trustworthy, no matter how well it’s protected.
while (!(full handcount) && !(strong evidence outcome is correct)) {
    examine more ballots
}
while (!(full handcount) && !(strong evidence outcome is correct)) {
    examine more ballots
}
if (full handcount) {
    handcount result is final
}
Elections should be conducted with human-readable paper ballots. Paper ballots form a body of evidence that is not subject to manipulation by faulty software or hardware and that can be used to audit and verify the results of an election. Human-readable paper ballots may be marked by hand or by machine (using a ballot-marking device), and they may be counted by hand or by machine (using an optical scanner), the report says. Voters should have an opportunity to review and confirm their selections before depositing the ballot for tabulation. Voting machines that do not provide the capacity for independent auditing – i.e., machines that do not produce a printout of a voter’s selections that can be verified by the voter and used in audits – should be removed from service as soon as possible.

States should mandate a specific type of audit known as a “risk-limiting” audit prior to the certification of election results. By examining a statistically appropriate random sample of paper ballots, risk-limiting audits can determine with a high level of confidence whether a reported election outcome reflects a correct tabulation.
Risk-Limiting Audits

- Endorsed by NASEM, PCEA, ASA, LWV, CC, VV, ...

- ~60 pilot audits in AK, CA, CO, GA, IN, KS, MI, MT, NJ, OH, OR, PA, RI, WA, WY, VA, DK.

- CA counties: Alameda, El Dorado, Humboldt, Inyo, Madera, Marin, Merced, Monterey, Napa, Orange, San Francisco, San Luis Obispo, Santa Clara, Santa Cruz, Stanislaus, Ventura, Yolo.


- Laws in CA, CO, GA, NV, NJ, OH, OR, RI, TX, VA, WA
Sets of Half-Average Nulls Generate Risk-Limiting Audits: SHANGRLA

Philip B. Stark
University of California, Berkeley, CA, USA
stark@stat.berkeley.edu

Abstract. Risk-limiting audits (RLAs) for many social choice functions can be reduced to testing sets of null hypotheses of the form “the average of this list is not greater than 1/2” for a collection of finite lists of nonnegative numbers. Such social choice functions include majority, super-majority, plurality, multi-winner plurality, Instant Runoff Voting (IRV), Borda count, approval voting, and STAR-Voting, among others. The audit stops without a full hand count if all the null hypotheses are rejected. The nulls can be tested in many ways. Ballot polling is particularly simple; two new ballot-polling risk-measuring functions for sampling without replacement are given. Ballot-level comparison audits transform each null into an equivalent assertion that the mean of re-scaled tabulation errors is not greater than 1/2. In turn, that null can then be tested using the same statistical methods used for ballot polling—applied to different finite lists of nonnegative numbers. The SHANGRLA approach thus reduces auditing different social choice functions and different audit methods to the same simple statistical problem. Moreover, SHANGRLA comparison audits are more efficient than previous comparison audits for two reasons: (i) for most social choice functions, the conditions tested are both necessary and sufficient for the reported outcome to be correct, while previous methods tested conditions that were sufficient but not necessary, and (ii) the tests avoid a conservative approximation. The SHANGRLA abstraction simplifies stratified audits, including audits that combine ballot polling with ballot-level comparisons, producing sharper audits than the “SURF” approach. SHANGRLA works with the “phantoms to evil zombies” strategy to treat missing ballot cards and missing or redacted cast vote records. That also facilitates sampling from “ballot-style manifests,” which can dramatically improve efficiency when the audited contests do not appear on every ballot card.
\( b_i \) is \( i \)th ballot card, \( N \) cards in all.

\[
1_{\text{candidate}}(b_i) := \begin{cases} 
1, & \text{ballot } i \text{ has a mark for candidate} \\
0, & \text{otherwise.}
\end{cases}
\]

\[
A_{\text{Alice,Bob}}(b_i) := \frac{1_{\text{Alice}}(b_i) - 1_{\text{Bob}}(b_i) + 1}{2} \in [0, 1].
\]

mark for Alice but not Bob, \( A_{\text{Alice,Bob}}(b_i) = 1 \).

mark for Bob but not Alice, \( A_{\text{Alice,Bob}}(b_i) = 0 \).

marks for both (overvote) or neither (undervote) or doesn’t contain contest, 
\( A_{\text{Alice,Bob}}(b_i) = 1/2 \).
$\bar{A}^b_{\text{Alice, Bob}} := \frac{1}{N} \sum_{i=1}^{N} A_{\text{Alice, Bob}}(b_i)$. 

Mean of a finite list of $N$ bounded numbers.

Alice won iff $\bar{A}^b_{\text{Alice, Bob}} > 1/2$. 
Plurality & Approval Voting

$K \geq 1$ winners, $C > K$ candidates in all.

Candidates $\{w_k\}_{k=1}^K$ are reported winners.

Candidates $\{\ell_j\}_{j=1}^{C-K}$ reported losers.
Plurality & Approval Voting

\[ K \geq 1 \text{ winners, } C > K \text{ candidates in all.} \]

Candidates \( \{w_k\}_{k=1}^{K} \) are reported winners.

Candidates \( \{\ell_j\}_{j=1}^{C-K} \) reported losers.

Outcome correct iff

\[ \bar{A}_{w_k,\ell_j}^b > 1/2, \quad \text{for all } 1 \leq k \leq K, \ 1 \leq j \leq C - K \]

\[ K(C - K) \text{ inequalities.} \]
Plurality & Approval Voting

\( K \geq 1 \) winners, \( C > K \) candidates in all.

Candidates \( \{ w_k \}_{k=1}^K \) are reported winners.

Candidates \( \{ \ell_j \}_{j=1}^{C-K} \) reported losers.

Outcome correct iff

\[
\bar{A}^{b}_{w_k, \ell_j} > 1/2, \quad \text{for all} \ 1 \leq k \leq K, \ 1 \leq j \leq C - K
\]

\( K(C - K) \) inequalities.

Same approach works for D’Hondt & other proportional representation schemes. (Stark & Teague 2015)
Super-majority

\( f \in (0, 1]. \)

Alice won iff

\[(\text{votes for Alice}) > f \times (\text{valid votes for anyone})\]

Set

\[A(b_i) := \begin{cases} 
\frac{1}{2f}, & b_i \text{ has a mark for Alice and no one else} \\
0, & b_i \text{ has a mark for exactly one candidate, not Alice} \\
\frac{1}{2}, & \text{otherwise.}
\end{cases}\]

Alice won iff

\[\overline{A}^b > 1/2.\]
Winner is the candidate who gets most “points” in total.

\[ s_{\text{Alice}}(b_i) : \text{Alice’s score on ballot } i. \]

\[ s_{\text{cand}}(b_i) : \text{another candidate’s score on ballot } i. \]

\[ s^+ : \text{upper bound on the score any candidate can get on a ballot.} \]

Alice beat the other candidate iff Alice’s total score is bigger than theirs:

\[ A_{\text{Alice},c}(b_i) := \frac{s_{\text{Alice}}(b_i) - s_{\text{cand}}(b_i) + s^+}{2s^+}. \]

Alice won iff \[ A^b_{\text{Alice},c} > 1/2 \] for every other candidate \( c \).
2 types of assertions (Blom et al. 2018):

1. Candidate \(i\) has more first-place ranks than candidate \(j\) has total mentions.
2. After a set of candidates \(E\) have been eliminated from consideration, candidate \(i\) is ranked higher than candidate \(j\) on more ballots than \textit{vice versa}.

Both can be written \(\bar{A}^b > 1/2\).

Finite set of such assertions implies reported outcome is right.

More than one set suffices; can optimize expected workload.
Test *complementary null hypothesis* $\bar{A}^b \leq 1/2$ sequentially.

- Audit until either all complementary null hypotheses about a contest are rejected at significance level $\alpha$ or until all ballots have been tabulated by hand.
- Yields a RLA of the contest in question at risk limit $\alpha$.
- No multiplicity adjustment needed.
Sequential tests (Wald, 1945) and martingales

Key object: nonnegative (super)martingale

Sequence of rvs \((Z_j), j = 1, \ldots\) s.t.

- \(\mathbb{E}|Z_j| < \infty\)
- \(\mathbb{E}(Z_{j+1}|Z_1, \ldots, Z_j) = (\leq)Z_j\)
- \(\mathbb{P}(Z_j \geq 0) = 1\)
Ville’s inequality (1939)

If \((Z_j)\) is a nonnegative supermartingale, then for any \(\alpha \in (0, 1]\) and all \(J \in \{1, \ldots, N\}\),

\[
\Pr \left( \max_{1 \leq j \leq J} Z_j \geq 1/\alpha \right) \leq \alpha \mathbb{E}|Z_J|.
\]
ALPHA: Audit that Learn from Previously Hand-Audited Ballots

Philip B. Stark
University of California, Berkeley
February 3, 2022

Abstract. A risk-limiting election audit (RLA) offers a statistical guarantee: if the reported electoral outcome is incorrect, the audit has at most a known maximum chance (the risk limit) of not correcting it before it becomes final. BRAVO [10], based on Wald’s sequential probability ratio test for the Bernoulli distribution, is the most widely tried method for RLAs. It has limitations. It cannot accommodate sampling without replacement or stratified sampling, which can improve efficiency and are sometimes required by law. It applies only to ballot-polling audits, which are less efficient than comparison audits. It applies to plurality, majority, super-majority, proportional representation, and ranked-choice voting contests, but not to many other social choice functions for which there are RLA methods, such as approval voting, STAR-voting, Borda count, and general scoring rules. And while BRAVO has the smallest expected sample size among sequentially valid ballot-polling-with-replacement methods when the reported vote shares are exactly correct, BRAVO can require arbitrarily large samples when the reported reported winner(s) really won but the reported vote shares are incorrect. ALPHA is a simple generalization of BRAVO that (i) works for sampling with and without replacement; (ii) can be used with stratified sampling; (iii) works not only for ballot-polling but also for ballot-level comparisons, batch-polling, and batch-level comparison audits, sampling with or without replacement, uniformly or with weights proportional to a measure of size; (iv) works for all social choice functions covered by SHANGRLA [19], including approval voting, Borda count, and all scoring rules; and (v) in situations where both ALPHA and BRAVO apply, requires smaller samples than BRAVO when the reported vote shares are wrong but the outcome is correct—five orders of magnitude in some examples. ALPHA includes the family of betting martingale tests in RlACS [27], with a different betting strategy parameterized as an estimator of the population mean and the flexibility to accommodate sampling weights and population bounds that change with each draw. A Python implementation is provided.
Test $\theta \leq \mu$ against the alternative $\theta > \mu$.

- $X^j := (X_1, \ldots, X_j); \ X_i \in [0, u_i]$.
- $\mu_j := \mathbb{E}(X_j|X^{j-1})$ computed under the null $\theta = \mu$.
- $\eta_j = \eta_j(X^{j-1}), j = 1, \ldots$, a predictable sequence: can’t depend on $X_k$ for $k \geq j$.

$T_0 := 1$;

$$T_j := T_{j-1}u_{j-1}^{-1}\left(X_j \frac{\eta_j}{\mu_j} + (u_j - X_j)\frac{u_j - \eta_j}{u_j - \mu_j}\right), \ j = 1, \ldots.$$  \hspace{2cm} (1)

($T_j$) is a nonnegative supermartingale w/ expected value $\leq 1$ if $\theta \leq \mu$.

Thus if $\theta \leq \mu$,

$$\mathbb{P}\left\{\max_j T_j \geq 1/\alpha\right\} \leq \alpha.$$
- Set audit parameters
  - risk limit \( \alpha \in (0, 1) \); \# cards \( N \), sampling method, \( u_i, \eta_0 \)
  - Pick \( \eta(i, X^{i-1}) \in (\mu_i, u_i] \), where \( \mu_i := \mathbb{E}(X_i|X^{i-1}) \) is computed under the null.

- Initialize variables
  - \( j \leftarrow 0 \): sample number
  - \( T \leftarrow 1 \): test statistic
  - \( S \leftarrow 0 \): sample sum
  - \( m = 1/2 \): population mean under the null

- While \( T < 1/\alpha \) and not all ballot cards have been audited:
  - draw a ballot card at random
  - \( j \leftarrow j + 1 \)
  - determine \( X_j \) by applying assorter to selected card
  - if \( m < 0 \), \( T \leftarrow \infty \); else \( T \leftarrow Tu_j^{-1} \left( X_j \frac{\eta(j, S)}{m} + (u_j - X_j) \frac{u - \eta(j, S)}{u - m} \right) \)
  - \( S \leftarrow S + X_j \)
  - if sampling w/o replacement, \( m \leftarrow (N/2 - S)/(N - j + 1) \)
  - if desired, break & conduct a full hand count
Use system’s interpretation of individual ballots or batches of ballots.
Like checking an expense report.

\( b_i \) is \( i \)th ballot, \( c_i \) is cast-vote record for \( i \)th ballot.

A an assor ter.

_overstatement error_ for \( i \)th ballot is

\[
\omega_i := A(c_i) - A(b_i) \leq A(c_i) \leq u,
\]

where \( u \) is an upper bound on the value \( A \) assigns to any ballot card or CVR.
$v := 2\bar{A}^c - 1$, *reported assorter margin*.

$B(b_i, c) := (1 - \omega_i/u)/(2 - v/u) > 0, \ i = 1, \ldots, N$.

$B$ assigns non-negative numbers to ballots.

Reported outcome correct iff

$$\bar{B} > 1/2.$$
Stratified sampling

Cast ballots are partitioned into $S \geq 2$ strata.

Stratum $s$ contains $N_s$ cast ballots.

Let $\bar{A}_s^b$ denote the mean of the assorter applied to just the ballot cards in stratum $s$. Then

$$\bar{A}^b = \frac{1}{N} \sum_{s=1}^{S} N_s \bar{A}_s^b = \sum_{s=1}^{S} \frac{N_s}{N} \bar{A}_s^b.$$  

Can reject the hypothesis $\bar{A}^b \leq 1/2$ if we can reject the hypothesis

$$\bigcap_{s \in S} \left\{ \frac{N_s}{N} \bar{A}_s^b \leq \beta_s \right\}$$

for all $(\beta_s)_{s=1}^{S}$ s.t. $\sum_{s=1}^{S} \beta_s \leq 1/2.$

Union-Intersection Test
Sample design

- individual ballots?
- groups of ballots?
- stratify? (law, logistics, equipment capabilities, ...)
- sampling probabilities?
- w/ replacement? w/o replacement? Bernoulli?
- fully sequential? escalation schedule?
Open research questions

- What is the class of social choice functions that can be audited with SHANGRLA?
- If there are sufficient conditions, are there also necessary and sufficient conditions?
- Are all sets of necessary and sufficient conditions equally expensive to audit?
- Can “round-by-round” sampling reduce sample sizes?
Wrinkles

- ~20% of U.S. voters don’t vote on paper
- States adopting universal-use BMDs: paper trail hackable/untrustworthy
- inadequate rules for chain of custody, ballot accounting, pollbook reconciliation, eligibility verification, ...
- need transparent high-quality randomness
  - public ceremony of die rolls, published crypto-quality PRNG
- missing ballots; imperfect manifests (Bañuelos & Stark 2012)
- producing CVRs linked to ballots while preserving vote anonymity; redacted CVRs
- preserve privacy but ensure the public can confirm audit didn’t stop too soon
Open-source software

- auditTools
- ballotPollTools
- SUITE
- SHANGRLA
- ALPHA
- Arlo
Evidence-Based Elections: 3 C's

- Voters *CREATE* complete, durable, verified audit trail.
Evidence-Based Elections: 3 C's

- Voters *CREATE* complete, durable, verified audit trail.
- LEO *CARES FOR* the audit trail adequately to ensure it remains complete and accurate.
Evidence-Based Elections: 3 C’s

- Voters *CREATE* complete, durable, verified audit trail.
- LEO *CARES FOR* the audit trail adequately to ensure it remains complete and accurate.
- Verifiable audit *CHECKS* reported results against the paper