Evidence-Based Elections

Philip B. Stark
9 June 2021

University of California, Berkeley
Many collaborators including (most recently) Andrew Appel, Josh Benaloh, Matt Bernhard, Michelle Blom, Andrew Conway, Rich DeMillo, Steve Evans, Amanda Glazer, Alex Halderman, Mark Lindeman, Kellie Ottoboni, Eddie Perez, Ron Rivest, Peter Ryan, Jake Spertus, Peter Stuckey, Vanessa Teague, Poorvi Vora
In Torrent of Falsehoods, Trump Claims Election Is Being Stolen

Most television networks cut away from the statement President Trump gave Thursday night from the White House briefing room on the grounds that what he was saying was not true.

“If you count the legal votes, I easily win,” President Trump said Thursday night in an unusually subdued, 17-minute televised statement from the lectern in the White House briefing room. Doug Mills/The New York Times
Half of Republicans say Biden won because of a 'rigged' election: Reuters/Ipsos poll

(Reuters) - About half of all Republicans believe President Donald Trump “rightfully won” the U.S. election but that it was stolen from him by widespread voter fraud that favored Democratic President-elect Joe Biden, according to a new Reuters/Ipsos opinion poll.

The Nov. 13-17 opinion poll showed that Trump’s open defiance of Biden’s victory in both the popular vote and Electoral College appears to be affecting the public's confidence in American democracy, especially among Republicans.
Is Trump right about Georgia vote?

By Robert Sanders, Media relations | NOVEMBER 13, 2020
**JOINT STATEMENT FROM ELECTIONS INFRASTRUCTURE GOVERNMENT COORDINATING COUNCIL & THE ELECTION INFRASTRUCTURE SECTOR COORDINATING EXECUTIVE COMMITTEES**

Original release date: November 12, 2020

WASHINGTON – The members of Election Infrastructure Government Coordinating Council (GCC) Executive Committee – Cybersecurity and Infrastructure Security Agency (CISA) Assistant Director Bob Kolasky, U.S. Election Assistance Commission Chair Benjamin Hovland, National Association of Secretaries of State (NASS) President Maggie Toulouse Oliver, National Association of State Election Directors (NASED) President Lori Augino, and Escambia County (Florida) Supervisor of Elections David Stafford – and the members of the Election Infrastructure Sector Coordinating Council (SCC) – Chair Brian Hancock (Unisyn Voting Solutions), Vice Chair Sam Derheimer (Hart InterCivic), Chris Wlaschin (Election Systems & Software), Ericka Haas (Electronic Registration Information Center), and Maria Bianchi (Democracy Works) - released the following statement:

"The November 3rd election was the most secure in American history. Right now, across the country, election officials are reviewing and double checking the entire election process prior to finalizing the result."

"When states have close elections, many will recount ballots. All of the states with close results in the 2020 presidential race have paper records of each vote, allowing the ability to go back and count each ballot if necessary. This is an added benefit for security and resilience. This process allows for the identification and correction of any mistakes or errors. There is no evidence that any voting system deleted or lost votes, changed votes, or was in any way compromised."

"Other security measures like pre-election testing, state certification of voting equipment, and the U.S. Election Assistance Commission's (EAC) certification of voting equipment help to build additional confidence in the voting systems used in 2020."

"While we know there are many unfounded claims and opportunities for misinformation about the process of our elections, we can assure you we have the utmost confidence in the security and integrity of our elections, and you should too. When you have questions, turn to elections officials as trusted voices as they administer elections.”
Trump supporters file lawsuit asking Georgia to decertify election, declare Trump the winner

Sidney Powell files voting lawsuit in Ga.
Sidney Powell shares 270-page binder of documents buttressing election fraud claims

by Daniel Chaitin, Breaking News Editor | December 27, 2020 08:56 PM
| Updated Dec 27, 2020, 10:33 PM
IN THE
SUPREME COURT OF THE UNITED STATES

L. LIN WOOD, JR.

Petitioner,

vs.

BRAD RAFFENSPERGER, et al.,

Respondents.

PETITION FOR WRIT OF CERTIORARI

On Petition for a Writ of Certiorari to the Eleventh Circuit Court of Appeals.

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FIFTH SUPPLEMENTAL DECLARATION OF PHILIP B. STARK

PHILIP B. STARK hereby declares as follows:

1. This statement supplements my declarations of September 9, 2018, September 30, 2018, October 22, 2019, and December 16, 2019. I stand by everything in the previous declarations.

I. False Assertions about the Fulton County Pilot Audit

2. Secretary of State Raffensperger issued the following (undated) press release on approximately June 30, 2020:1

AUDIT SUPPORTS PRIMARY OUTCOME

(ATLANTA) – A pilot post-election audit Monday confirmed the outcomes of the presidential preference primaries in Fulton County, Secretary of State Brad Raffensperger announced today.

“This procedure demonstrates once again the validity of the results produced by Georgia’s new secure paper-ballot system,” [SOS Raffensperger] said. “Auditing

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Lin Wood Doxed Georgia Officials to Hundreds of Thousands of QAnon Supporters

The pro-Trump lawyer asked an “Army of Patriots” on Telegram to dig up dirt on officials who will decide whether he is disbarred or not.

Pro-Trump lawyer and major QAnon booster Lin Wood has urged hundreds of thousands of his supporters to dig up dirt on Georgia officials who will decide if he should be disbarred or not — and to help them in their research, Wood published the officials' addresses on social media.

In a 1,600-page filing, the State Disciplinary Board of the State Bar of Georgia said that it had “received information concerning the above-named attorney that suggests that said attorney may have violated one or more of the Georgia Rules of Professional Conduct.”

MORE LIKE THIS

Georgia’s New QAnon Congresswoman Refused to Wear a Mask at Her Swearing-in

The Democrats’ Lazy QAnon Attack Ad Will Only Make Things Worse

Majorie Taylor Greene Believes in Fracciokship, QAnon’s Wildest Conspiracy Theory
Pro-Trump Lawyer Lin Wood Is Investigated for Alleged Illegal Voting in Georgia

Wood promoted claims that the election was rigged against former President Donald Trump

Lin Wood said he decided on Monday that he would change his residency to South Carolina, and that he has lived in Georgia since 1955.

PHOTO: BEN MARGOT/ASSOCIATED PRESS

By Alexa Corse and Erin Ailworth
Feb. 2, 2021 11:43 pm ET
Mike Lindell, Mary Fanning, and Brannon Howse Present the Docu-movie:

**ABSOULTE PROOF**

Exposing Election Fraud and the Theft of America by Enemies Foreign and Domestic
Republicans Are P-Hacking the Supreme Court

Texas is seeking to overturn the 2020 election based on a shoddy statistical analysis. It’s just what you would expect from medical researchers.

I SPENT THE last month watching, with alternating apprehension and delight, as President Trump’s cynical legal efforts to overturn the presidential election deteriorated into absurdity. After dozens of lawsuits were thrown out of court, and votes were certified in contested states, I thought we’d reached the end of the road. But it turns out there was one gut punch left to deliver, a bright red line no science-minded person like myself can bear to see crossed. That’s right, Donald Trump misused statistics.

The Texas attorney general filed a lawsuit Monday asking the US Supreme Court to intervene in the election. Before your heart rhythm changes too dramatically, I should tell you that legal experts consider the case “doomed.” That doesn’t mean the lawsuit can’t be dangerous. It introduced the strange-but-real number “quadrillion” into the political discourse for a couple of news cycles and seeded a new set of numerical conspiracy theories that could live on for years as so-called proof of election fraud.

On Tuesday, as 18 more states prepared to back the Texas lawsuit, press secretary Kayleigh McEnany tweeted out one of its central claims: “Chances of Biden winning Pennsylvania, Michigan, Georgia, Wisconsin independently alter @realDonaldTrump’s early lead is less than one in a quadrillion.” She then proceeded to type out the number with all of its 15 glorious zeroes.
WASHINGTON – The Supreme Court on Monday formally rejected a handful of cases related to the 2020 election, including disputes from Pennsylvania that had divided the justices just before the election.

The cases the justices rejected involved election challenges filed by former President Donald Trump and his allies in five states President Joe Biden won: Arizona, Georgia, Michigan, Pennsylvania and Wisconsin.

Other than the disputes from Pennsylvania, the justices' decision not to hear the cases was unsurprising. The court had previously taken no action in those cases and in January had turned away pleas that the cases be fast-tracked, again suggesting the justices were not interested in hearing them.
Sidney Powell’s secret ‘military intelligence expert,’ key to fraud claims in election lawsuits, never worked in military intelligence
Sidney Powell Drops Georgia Suit, Marking End to Presidential Election-Related Lawsuits in State

BY NICOLE FALLERT ON 1/19/21 AT 5:00 PM EST

Politicians And Celebrities React To Georgia Senate Election Results As Democrats Take Control Of Senate
Dominion sues pro-Trump lawyer Sidney Powell, seeking more than $1.3 billion
Sidney Powell's legal defense: 'Reasonable people' wouldn't believe her election fraud claims

Lawyers for the Trump ally claim she was just sharing an opinion when she said the election was stolen using machines built to rig races for Hugo Chavez.
Politics
Dominion sues Giuliani over false election fraud claims
January 26, 2021 | 12:51 PM PST

Voting machine company Dominion filed a $1.3 billion lawsuit against former president Donald Trump’s lawyer Rudy Giuliani on Jan. 25.

Related
Giuliani wasn’t just a Trump partisan but a shrewd marketer of vitamins, gold, lawsuit says
Dominion Sues MyPillow, CEO Mike Lindell Over Election Claims

The voting-machine maker’s lawsuit alleges defamation, seeks more than $1.3 billion in damages

By Alexa Corpse
Updated Feb 22, 2021 8:02 am ET

WASHINGTON—One of the largest makers of voting machines in the U.S. on Monday sued a prominent supporter of former President Donald Trump, alleging that the businessman had defamed the company with false accusations that it had rigged the 2020 election for Joe Biden.
Why Georgia’s Unscientific Recount ‘Horrified’ Experts

Observers, including the inventor of the auditing process used by the state, were skeptical of a measure seemingly aimed at placating the GOP.

By Timothy Pratt
- US elections neither *tamper evident* nor *resilient*
- US elections neither *tamper evident* nor *resilient*

- Need systems/procedures that can provide strong evidence that the reported winners really won.
Elections Should be Grounded in Evidence, Not Blind Trust

President Donald Trump’s attempt to pressure Georgia election officials to “find” votes he didn’t win is keeping election integrity in the spotlight. Tomorrow’s Senate runoffs will determine which party controls the chamber, and there’s a high likelihood that this round of voting will also be declared illegitimate by the losers. Even though there is no compelling evidence the 2020 vote was rigged, U.S. elections are insufficiently equipped to counter such claims because of a flaw in American voting. The way we conduct elections does not routinely produce public evidence that outcomes are correct.
• Voters hand-mark paper ballots to create a trustworthy, durable paper vote record. Voters who cannot hand-mark a ballot independently are provided assistive technologies, such as electronic ballot marking devices. But because these devices are subject to hacking, bugs, and software misconfiguration, the use of such ballot-marking devices should be limited.

• Election officials protect the paper ballots to ensure no ballot has been added, removed, or altered. This requires stringent physical security protocols and ballot accounting, among other things.

• Election officials count the votes, using technology if they choose. If the technology altered the outcome, that will (with high confidence) be corrected by the steps below.

• Election officials reconcile and verify the number of ballots and the number of voters, with a complete canvass to ensure that every validly cast ballot is included in the count.

• Election officials check whether the paper trail is trustworthy using a transparent “compliance audit,” reviewing chain-of-custody logs and security video, verifying voter eligibility, reconciling numbers of ballots of each style against poll book signatures and other records, and accounting for every ballot that was issued.

• Election officials check the results with an audit that has a known, large probability of catching and correcting wrong reported outcomes—and no chance of altering correct outcomes. The inventory of paper ballots used in the audit must be complete and the audit must inspect the original hand-marked ballots, not images or copies.
None of these steps stands alone. An unexamined set of paper ballots, no matter how trustworthy, provides no evidence. Conversely, no matter how rigorous, audits and recounts of an untrustworthy paper trail provide no evidence that the reported winners won. Auditing or recounting machine-marked ballots or hand-marked ballots that have not been kept secure can check whether the reported outcome reflects that paper trail, but cannot provide evidence that the reported winners won.
Security properties of paper

- tangible/accountable
- tamper evident
- human readable
- large alteration/substitution attacks require physical access & many accomplices
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Not all paper is trustworthy
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2. In his testimony on 11 September 2020, Defendant’s expert Dr. Ben Adida made a

A just-released study says over ninety percent of errors introduced by ballot marking devices go undetected.
Serious design flaw in ESS ExpressVote touchscreen: “permission to cheat”

SEPTEMBER 14, 2018 BY ANDREW APPEL

Kansas, Delaware, and New Jersey are in the process of purchasing voting machines with a serious design flaw, and they should reconsider while there is still time!

Over the past 15 years, almost all the states have moved away from paperless touchscreen voting systems (DREs) to optical-scan paper ballots. They’ve done so because if a paperless touchscreen is hacked to give fraudulent results, there’s no way to know and no way to correct; but if an optical scanner were hacked to give fraudulent results, the fraud could be detected by a random audit of the paper ballots that the voters actually marked, and corrected by a recount of those paper ballots.
Donald Trump’s Favorite Voting Machines

Ballot-marking devices in key swing states could give him the perfect excuse to contest the election

by Art Levine  September 23, 2020  POLITICS
Ballot-Marking Devices (BMDs) Cannot Assure the Will of the Voters

Andrew W. Appel\textsuperscript{1} \hspace{1cm} Richard A. DeMillo\textsuperscript{1}
Princeton University \hspace{1cm} Georgia Tech

Philip B. Stark\textsuperscript{1}
Univ. of California, Berkeley

February 14, 2020

Abstract

The complexity of U.S. elections usually requires computers to count ballots—but computers can be hacked, so election integrity requires a voting system in which paper ballots can be recounted by hand. However, paper ballots provide no assurance unless they accurately record the votes as expressed by the voters.

Voters can express their intent by indelibly hand-marking ballots, or using computers called ballot-marking device (BMDs). Voters can make mistakes in expressing their intent in either technology, but only BMDs are also subject to hacking, bugs, and misconfiguration of the software that prints the marked ballots. Most voters do not review BMD-printed ballots, and those who do often fail to notice when the printed vote is not what they expressed on the touchscreen. Furthermore, there is no action a voter can take to demonstrate to election officials that a BMD altered their expressed votes, nor is there a corrective action that election officials can take if notified by voters—there is no way to deter, contain, or correct computer hacking in BMDs. These are the essential security flaws of BMDs.
Testing Cannot Tell Whether Ballot-Marking Devices Alter Election Outcomes

Philip B. Stark and Ran Xie

1 University of California, Berkeley
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29 July 2020

Abstract. Like all computerized systems, ballot-marking devices (BMDs) can be hacked, misprogrammed, and misconfigured. BMD printout might not reflect what the BMD screen or audio conveyed to the voter. If voters complain that BMDs misbehaved, officials have no way to tell whether BMDs malfunctioned, the voters erred, or the voters are attempting to cast doubt on the election. Several approaches to testing BMDs have been proposed. In pre-election logic and accuracy (L&A) tests, trusted agents input known test patterns into the BMD and check whether the printout matches. In parallel or live testing, trusted agents use the BMDs on election day, emulating voters. In passive testing, trusted agents monitor the rate at which voters “spoil” ballots and request another opportunity to mark a ballot; an anomalously high rate might result from BMD malfunctions. In practice, none of these methods can protect against outcome-altering problems. L&A testing is ineffective against malware in part because BMDs “know” the time and date of the test and the election. Neither L&A nor parallel testing can probe even a small fraction of the combinations of voter preferences, device settings, ballot language, duration of voter interaction, input and output interfaces, and other variables that could comprise enough votes to change outcomes. Under mild assumptions, to develop a model of voter interactions with BMDs accurate enough to ensure that parallel tests could reliably detect changes to 0% of the votes (which could change margins by 10% or more) would require monitoring the behavior of more than a million voters in each jurisdiction in minute detail—but the median turnout by jurisdiction in the U.S. is under 3000 voters, and 2/3 of U.S. jurisdictions have fewer than 43,000 active voters. Moreover, all voter privacy would be lost. Given an accurate model of voter behavior, the number of tests required is still larger than the turnout in a typical U.S. jurisdiction. Even if less testing sufficed, it would require extra BMDs, new infrastructure for creating test interactions and reporting test results, additional polling-place staff, and more training. Under optimistic assumptions, passive testing that has a 99% chance of detecting a 1% change to the margin with a 1% false alarm rate is impossible in jurisdictions with fewer than about 1 million voters, even if the “normal” spoiled ballot rate were known exactly and did not vary from election to election and place to place. Passive testing would also require training and infrastructure to monitor the spoiled ballot rate in real time. And if parallel or passive testing discovers a problem, the only remedy is a new election: there is no way to reconstruct the correct election result from an untrustworthy paper trail. Minimizing the number of votes cast using BMDs is prudent election administration.
- Hand-marked paper ballots are a record of what the voter did.
- Machine-marked paper ballots are a record of what the machine did.
- BMDs make voters responsible for catching & correcting machine errors/bugs/hacks
- Few voters notice errors in BMD printout
Did the reported winner really win?

- Procedure-based vs. evidence-based elections
  - sterile scalpel v. patient’s condition
Did the reported winner really win?

- Procedure-based vs. evidence-based elections
  - sterile scalpel v. patient’s condition
- Any way of counting votes can make mistakes
- Every electronic system is vulnerable to bugs, configuration errors, & hacking
- Did error/bugs/hacking cause losing candidate(s) to appear to win?
- Minimum accuracy standard: find who really won.
Voting system properties needed to justify public trust

- (Strong) Software Independence
- Contestability
- Defensibility
Voting system properties needed to justify public trust

- (Strong) Software Independence
- Contestability
- Defensibility

DREs, BMDs, online voting are none of the above.
Evidence-Based Elections

P.B. Stark and D.A. Wagner

Abstract—We propose an alternative to current requirements for certifying voting equipment and conducting elections. We argue that elections should be structured to provide convincing affirmative evidence that the reported outcomes actually reflect how people voted. This can be accomplished with a combination of software-independent voting systems, compliance audits, and risk-limiting audits. Together, these yield a resilient canvass framework: a fault-tolerant approach to conducting elections that gives strong evidence that the reported outcome is correct or reports that the evidence is not convincing. We argue that, if evidence-based elections are adopted, certification and testing of voting equipment can be relaxed, saving money and time and reducing barriers to innovation in voting systems—and election integrity will benefit. We conclude that there should be more regulation of the evidence trail and less regulation of equipment, and that compliance audits and risk-limiting audits should be required.

Keywords: elections, software-independent voting system, risk-limiting audit, resilient canvass framework EDICS SEC-INTE, APP-CRIM, APP-INTE, APP-OTHE.

I. INTRODUCTION

Ideally, what should an election do? Certainly, an election should find out who won, but we believe it also should produce convincing evidence that it found the real winners—or report that it cannot. This is not automatic; it requires thoughtful design of voting equipment, carefully planned and implemented voting and vote counting processes, and rigorous post-election auditing.

While approximately 75% of US voters currently vote on equipment that produces a voter-verifiable paper record of the vote, about 25% vote on paperless electronic voting machines that do not produce such a record [1].

Because paperless electronic voting machines rely upon complex software and hardware, and because there is no feasible way to ensure that the voting software is free of bugs or that the hardware is executing the proper software, there is no guarantee that electronic voting machines record the voter’s votes accurately. And, because paperless voting machines preserve only an electronic record of the vote that cannot be directly observed by voters, there is no way to produce convincing evidence that the electronic record accurately reflects the voters’ intent. Internet voting shares the shortcomings of paperless electronic voting machines, and has additional vulnerabilities.

Numerous failures of electronic voting equipment have been documented. Paperless voting machines in Carteret County, North Carolina irrevocably lost 4,400 votes; other machines in Mecklenburg, North Carolina recorded 3,955 more votes than the number of people who voted; in Bernaalillo County, New Mexico, machines recorded 2,700 more votes than voters; in Mahoning County, Ohio, some machines reported a negative total vote count; and in Fairfax, Virginia, county officials found that for every hundred or so votes cast for one candidate, the electronic voting machines subtracted one vote for her [2]. In short, when elections are conducted on paperless voting equipment, there is no guarantee of voter intent and no assurance of the accuracy of the results.
Risk-Limiting Audits (RLAs, Stark, 2008)

- If there's a trustworthy paper record of votes, can check whether reported winner really won.
- If you accept a controlled “risk” of not correcting the reported outcome if it is wrong, typically don’t need to look at many ballots if outcome is right.
A risk-limiting audit has a known minimum chance of correcting the reported outcome if the reported outcome is wrong (& doesn’t alter correct outcomes).
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*Risk limit:* largest possible chance of *not* correcting reported outcome, if reported outcome is wrong.
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*Wrong* means accurate handcount of *trustworthy* paper would find different winner(s).
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*Risk limit*: largest possible chance of *not* correcting reported outcome, if reported outcome is wrong.

Wrong means accurate handcount of *trustworthy* paper would find different winner(s).

Establishing whether paper trail is trustworthy involves other processes, generically, *compliance audits*
while (!(full handcount) && !(strong evidence outcome is correct)) {
    examine more ballots
}
while (!full handcount) && !(strong evidence outcome is correct)) {
    examine more ballots
}
if (full handcount) {
    handcount result is final
}
Elections should be conducted with human-readable paper ballots. Paper ballots form a body of evidence that is not subject to manipulation by faulty software or hardware and that can be used to audit and verify the results of an election. Human-readable paper ballots may be marked by hand or by machine (using a ballot-marking device), and they may be counted by hand or by machine (using an optical scanner), the report says. Voters should have an opportunity to review and confirm their selections before depositing the ballot for tabulation. Voting machines that do not provide the capacity for independent auditing – i.e., machines that do not produce a printout of a voter’s selections that can be verified by the voter and used in audits – should be removed from service as soon as possible.

States should mandate a specific type of audit known as a “risk-limiting” audit prior to the certification of election results. By examining a statistically appropriate random sample of paper ballots, risk-limiting audits can determine with a high level of confidence whether a reported election outcome reflects a correct tabulation
Risk-Limiting Audits

- Endorsed by NASEM, PCEA, ASA, LWV, CC, VV, ...

- ~60 pilot audits in AK, CA, CO, GA, IN, KS, MI, MT, NJ, OH, OR, PA, RI, WA, WY, VA, DK.

- CA counties: Alameda, El Dorado, Humboldt, Inyo, Madera, Marin, Merced, Monterey, Napa, Orange, San Francisco, San Luis Obispo, Santa Clara, Santa Cruz, Stanislaus, Ventura, Yolo.


- Laws in CA, CO, RI, VA, WA
Role of math/stat

- Reduce workload!
- Get evidence about the population of cast ballots from a random sample.
- Guarantee a large chance of correcting wrong outcomes; minimize work if the outcome is correct.
- When can you stop inspecting ballots?
  - When there’s strong evidence that a full hand count is pointless
RLA as a hypothesis test

- Null hypothesis: reported outcome is *wrong*.
- Significance level (Type I error rate) is “risk”
- Frame the hypothesis quantitatively: necessary and sufficient conditions
$b_i$ is $i$th ballot card, $N$ cards in all.

$$1_{\text{candidate}}(b_i) := \begin{cases} 
1, & \text{ballot } i \text{ has a mark for candidate} \\
0, & \text{otherwise.}
\end{cases}$$

$$A_{\text{Alice,Bob}}(b_i) := \frac{1_{\text{Alice}}(b_i) - 1_{\text{Bob}}(b_i) + 1}{2} \in [0, 1].$$

Mark for Alice but not Bob, $A_{\text{Alice,Bob}}(b_i) = 1$.

Mark for Bob but not Alice, $A_{\text{Alice,Bob}}(b_i) = 0$.

Marks for both (overvote) or neither (undervote) or doesn't contain contest, $A_{\text{Alice,Bob}}(b_i) = 1/2$. 
\[ \overline{A}_{Alice, Bob}^b := \frac{1}{N} \sum_{i=1}^{N} A_{Alice, Bob}(b_i). \]

Mean of a finite nonnegative list of \( N \) numbers.

Alice won iff \( \overline{A}_{Alice, Bob}^b > 1/2. \)
$K \geq 1$ winners, $C > K$ candidates in all.

Candidates $\{w_k\}_{k=1}^K$ are reported winners.

Candidates $\{\ell_j\}_{j=1}^{C-K}$ reported losers.
Plurality & Approval Voting

$K \geq 1$ winners, $C > K$ candidates in all.

Candidates $\{w_k\}_{k=1}^K$ are reported winners.

Candidates $\{\ell_j\}_{j=1}^{C-K}$ reported losers.

Outcome correct iff

$$\bar{A}_{w_k,\ell_j}^b > 1/2, \quad \text{for all } 1 \leq k \leq K, \ 1 \leq j \leq C - K$$

$K(C - K)$ inequalities.
Plurality & Approval Voting

\[ K \geq 1 \text{ winners, } C > K \text{ candidates in all.} \]

Candidates \( \{w_k\}_{k=1}^{K} \) are reported winners.

Candidates \( \{\ell_j\}_{j=1}^{C-K} \) reported losers.

Outcome correct iff

\[ \bar{A}_{w_k,\ell_j}^b > 1/2, \quad \text{for all } 1 \leq k \leq K, \ 1 \leq j \leq C - K \]

\( K(C - K) \) inequalities.

Same approach works for D'Hondt & other proportional representation schemes. (Stark & Teague 2015)
Super-majority

\[ f \in (0, 1). \]

Alice won iff

\[
(votes\ for\ Alice) > f \times ((valid\ votes\ for\ Alice) + (valid\ votes\ for\ everyone\ else))
\]

Set

\[
A(b_i) := \begin{cases} 
\frac{1}{2f}, & b_i \ has \ a \ mark \ for \ Alice \ and \ no \ one \ else \\
0, & b_i \ has \ a \ mark \ for \ exactly \ one \ candidate, \ not \ Alice \\
\frac{1}{2}, & \text{otherwise.}
\end{cases}
\]

Alice won iff

\[
\bar{A}^b > \frac{1}{2}.
\]
Winner is the candidate who gets most “points” in total.

$s_{\text{Alice}}(b_i)$: Alice’s score on ballot $i$.

$s_{\text{cand}}(b_i)$: another candidate’s score on ballot $i$.

$s^+$: upper bound on the score any candidate can get on a ballot.

Alice beat the other candidate iff Alice’s total score is bigger than theirs:

$$A_{\text{Alice},c}(b_i) := \frac{s_{\text{Alice}}(b_i) - s_{\text{cand}}(b_i) + s^+}{2s^+}.$$

Alice won iff $\bar{A}_{\text{Alice},c} > 1/2$ for every other candidate $c$. 


2 types of assertions together give sufficient (not necessary) conditions (Blom et al. 2018):

1. Candidate $i$ has more first-place ranks than candidate $j$ has total mentions.
2. After a set of candidates $E$ have been eliminated from consideration, candidate $i$ is ranked higher than candidate $j$ on more ballots than *vice versa*.

Both can be written $\tilde{A}^b > 1/2$.

Finite set of such assertions implies reported outcome is right.

More than one set suffices; can optimize expected workload.
Test complementary null hypothesis $\bar{A}^b \leq 1/2$ sequentially.

- Audit until either all complementary null hypotheses about a contest are rejected at significance level $\alpha$ or until all ballots have been tabulated by hand.

- Yields a RLA of the contest in question at risk limit $\alpha$.

- No multiplicity adjustment needed.
Sequential testing originated w/ Wald (1945; military secret before).

Key object: martingale.

Sequence of rvs \{Z_j\} s.t.

- \(E|Z_j| < \infty\)
- \(E(Z_{j+1}|Z_1, \ldots, Z_j) = Z_j\)
If \( \{Z_j\} \) is a nonnegative martingale, then for any \( p > 0 \) and all \( J \in \{1, \ldots, N\} \),

\[
\Pr \left( \max_{1 \leq j \leq J} Z_j(t) > 1/p \right) \leq p \mathbb{E}|Z_J|.
\]

Markov’s inequality applied to optionally stopped martingales.
For \( j = 1, 2, \ldots \), let \( P_{j0} \) be the probability of \( X_1, \ldots, X_j \) under \( H_0 \); \( P_{j1} \) be the probability of \( X_1, \ldots, X_j \) under \( H_1 \).

\[
Z_j = \frac{P_{j1}}{P_{j0}}, \quad j = 1, 2, \ldots
\]

is a nonnegative martingale if \( H_0 \) is true.

\( 1/Z_j \) is a valid \( P \)-value for \( H_0 \) at step \( j \).
Ballot-polling audits

Sample sequentially w/o replacement from a finite population of $N$ non-negative items, \(\{x_1, \ldots, x_N\}\), with $x_j \geq 0$, $\forall j$.

Total is $N\bar{x} \geq 0$. Value of the $j$th item drawn is $X_j$.

If $\bar{x} = t$, $\mathbb{E}X_1 = t$, so $\mathbb{E}(X_1/t) = 1$.

Given $X_1, \ldots, X_n$, the total of the remaining $N - n$ items is $Nt - \sum_{j=1}^{n} X_j$, so the mean of the remaining items is

$$t_n := \frac{Nt - \sum_{j=1}^{n} X_j}{N - n} = \frac{t - \frac{1}{N} \sum_{j=1}^{n} X_j}{1 - n/N}.$$
Define

\[ Y_1(t) := \begin{cases} 
X_1/t, & Nt > 0, \\
1, & Nt = 0,
\end{cases} \]

and for \(1 \leq n \leq N - 1\),

\[ Y_{n+1}(t) := \begin{cases} 
X_{n+1} \frac{1 - \frac{n}{N}}{t - \frac{1}{N} \sum_{j=1}^{n} X_j}, & \sum_{j=1}^{n} X_j < Nt, \\
1, & \sum_{j=1}^{n} X_j \geq Nt.
\end{cases} \]

Then \(\mathbb{E}(Y_{n+1}(t) | Y_1, \ldots, Y_n) = 1\).
Let $Z_n(t) := \prod_{j=1}^{n} Y_j(t)$.

$E|Z_k| \leq \max_j x_j < \infty$ and

$$E(Z_{n+1}(t)|Z_1(t), \ldots, Z_n(t)) = E(Y_{n+1}(t)Z_n(t)|Z_1(t), \ldots, Z_n(t)) = Z_n(t).$$

Thus

$$(Z_1(t), Z_2(t), \ldots, Z_N(t))$$

is a non-negative closed martingale.

Thus a $P$-value for the hypothesis $\bar{x} = t$ for data $X_1, \ldots, X_J$ is $(\max_{1\leq j\leq J} Z_j(t))^{-1} \wedge 1$. 

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Many other martingales

Kaplan’s martingale (KMART)

Let \( S_j := \sum_{k=1}^{j} X_k \), \( \tilde{S}_j := S_j/N \), and \( \tilde{j} := 1 - (j - 1)/N \). Define

\[
Y_n := \int_0^1 \prod_{j=1}^{n} \left( \gamma \left[ X_j \frac{\tilde{j}}{t - \tilde{S}_{j-1}} - 1 \right] + 1 \right) d\gamma.
\]

Polynomial in \( \gamma \) of degree at most \( n \), with constant term 1.

Under the null, \( (Y_j)_{j=1}^{N} \) is a non-negative closed martingale with expected value 1.
Betting martingales (Waudby-Smith & Ramdas)

\[ Y_n(t) := \prod_{k=1}^{n} \left( 1 + \lambda_k(t_n)(X_k - t_k) \right) \]

where \( \lambda_k(t) \) a predictable sequence s.t. \( \lambda_k(t) \in [- (1 - t_k)^{-1}, (t_k)^{-1}] \).

Many good strategies for selecting \( \lambda_k \).
Use cast vote records (CVRs): system’s interpretation of each ballot.

Like checking an expense report.

$b_i$ is $i$th ballot, $c_i$ is cast-vote record for $i$th ballot.

$A$ an assorter.

*overstatement error* for $i$th ballot is

$$\omega_i := A(c_i) - A(b_i) \leq A(c_i) \leq u,$$

where $u$ is an upper bound on the value $A$ assigns to any ballot card or CVR.
\( v := 2\bar{A}^c - 1 \), reported assorter margin.

\( B(b_i, c) := \frac{1 - \omega_i/u}{2 - v/u} > 0, \quad i = 1, \ldots, N. \)

\( B \) assigns non-negative numbers to ballots.

Reported outcome correct iff

\[ \bar{B} > 1/2. \]
Stratified sampling

Cast ballots are partitioned into $S \geq 2$ strata.

Stratum $s$ contains $N_s$ cast ballots.

Let $\bar{A}_s^b$ denote the mean of the assorter applied to just the ballot cards in stratum $s$. Then

$$\bar{A}^b = \frac{1}{N} \sum_{s=1}^S N_s \bar{A}_s^b = \sum_{s=1}^S \frac{N_s}{N} \bar{A}_s^b.$$ 

Can reject the hypothesis $\bar{A}^b \leq 1/2$ if we can reject the hypothesis

$$\bigcap_{s \in S} \left\{ \frac{N_s}{N} \bar{A}_s^b \leq \beta_s \right\}$$

for all $(\beta_s)_{s=1}^S$ s.t. $\sum_{s=1}^S \beta_s \leq 1/2$.

Union-Intersection Test
Fisher’s Combining Function

\[ \{ P_s(\beta_s) \}_{s=1}^S \] are independent random variables.

If \( \bigcap_{s \in S} \left\{ \frac{N_s}{N} A_s^b \leq \beta_s \right\} \), distribution of

\[ -2 \sum_{s=1}^S \ln P_s(\beta_s) \]

is dominated by chi-square distribution with \( 2S \) degrees of freedom.

Low-dimensional optimization problem to maximize \( P \)-value over \( (\beta_s)_{s=1}^S \).
Sample design

- individual ballots?
- clusters of ballots?
- stratify? (logistics, equipment capabilities, ...)
- sampling probabilities?
- with replacement? without replacement? Bernoulli?
- fully sequential? batch-oriented?
Bayesian election audits

Limit the *upset probability*, the posterior probability that the reported outcome is wrong, given the sample, for a particular prior distribution on outcomes.
Bayesian election audits

Limit the *upset probability*, the posterior probability that the reported outcome is wrong, given the sample, for a particular prior distribution on outcomes.

Typically use Dirichlet-multinomial prior.

“Non-partisan” priors invariant under permutations of the candidate names.
A Bayesian Method for Auditing Elections

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Abstract

We propose an approach to post-election auditing based on Bayesian principles, and give experimental evidence for its efficiency and effectiveness. We call such an audit a “Bayes audit”. It aims to control the probability of miscertification (certifying a wrong election outcome). The miscertification probability is computed using a Bayesian model based on information gathered by the audit so far.

A Bayes audit is a single-ballot audit method applicable to any voting system (e.g. plurality, approval, IRV, Borda, Schulze, etc.) as long as the number of ballot types is not too large. The method requires only the ability to randomly sample single ballots and the ability to compute the election outcome for a profile of ballots. A Bayes audit does not require the computation of a “margin of victory” in order to get started.

1 Introduction

This section provides a quick introduction to post-election audits and our notation. Section 2 then presents our proposed Bayes audit procedure. Section 3 gives the results of our initial experiments using this method on simulated and real election data. Section 4 considers some extensions and variations of the basic method, and Sections 5 and 6 discuss and summarize what we have learned about the Bayes audit. Appendix A provides some additional technical details on efficient implementation methods.

1.1 Post-election audits

Informally, the purpose of a post-election audit is to check that the reported election outcome is correct, by auditing enough randomly chosen ballots.

Absolute certainty isn’t required of an audit (the only
...
Bayes/Frequentist duality

**Risk** of an audit for a set of cast votes and a reported outcome:

- probability of not correcting outcome, if reported outcome is wrong for that set of votes
- 0, if reported outcome is correct for that set of votes
Bayes/Frequentist duality

*Risk* of an audit for a set of cast votes and a reported outcome:

- probability of not correcting outcome, if reported outcome is wrong for that set of votes
- 0, if reported outcome is correct for that set of votes
- RLAs control *maximum* risk.
- Bayesian audits (Rivest & Shen) control *weighted average* of the risk. The prior sets the weights in the average.
- For 2-candidate plurality contest w/ no invalid votes, least-favorable prior has point mass 1/2 at tie, remaining 1/2 mass arbitrary over winning outcomes (Vora, 2018).
Wrinkles

- ~20% of U.S. voters don’t vote on paper
- ballot-marking devices make the paper trail hackable: current suit in GA
- inadequate rules for chain of custody, ballot accounting, pollbook reconciliation, signature verification, ...
- transparent high-quality randomness
  - public ceremony of die rolls, published crypto-quality PRNG
- missing ballots; imperfect manifests
  - “Manifest Phantoms to Evil Zombies”
- ability to produce CVRs linked to ballots
- redacted CVRs
Open-source software

- auditTools
- ballotPollTools
- SUITE
- SHANGRLA
- Arlo
Evidence-Based Elections: 3 C's

- Voters *CREATE* complete, durable, verified audit trail.
Evidence-Based Elections: 3 C’s

- Voters *CREATE* complete, durable, verified audit trail.
- LEO *CARES FOR* the audit trail adequately to ensure it remains complete and accurate.
Evidence-Based Elections: 3 C’s

- Voters *CREATE* complete, durable, verified audit trail.
- LEO *CARES FOR* the audit trail adequately to ensure it remains complete and accurate.
- Verifiable audit *CHECKS* reported results against the paper.
MIT News

MIT researchers identify security vulnerabilities in voting app
Mobile voting application could allow hackers to alter individual votes and may pose privacy issues for users.

Abby Abazorius | MIT News Office
February 13, 2020

In recent years, there has been a growing interest in using internet and mobile technology to increase access to the voting process. At the same time, computer security experts caution that paper ballots are the only secure means of voting.

Now, MIT researchers are raising another concern: They say they have uncovered security vulnerabilities in a mobile voting application that was used during the 2018 midterm elections in West Virginia. Their security analysis of the application, called Voatz, pinpoints a number of weaknesses, including the opportunity for hackers to alter, stop, or expose how an individual user has voted. Additionally, the researchers found that Voatz’s

MIT researchers have identified security flaws in a mobile voting application that allowed some overseas and military citizens to vote remotely, reports Lydia Emmanouilidou for PRI’s The World. “When things are opaque — when you can’t verify, when you can’t see what the code is doing — you can’t trust the outcome,” says graduate student Michael Specter, “there is no way of vetting that it’s doing the right thing.”
HackerOne Drops Mobile Voting App Vendor Voatz

Bug bounty platform provider cited "Voatz's pattern of interactions with the research community" in its decision to halt the app vendor's vulnerability disclosure program on HackerOne.

Mobile voting application vendor Voatz has been dismissed from HackerOne's bug bounty program platform, according to a report on CyberScoop.

Voatz — whose mobile voting app used in limited elections in a handful of states, including West Virginia and Colorado — has been under intense scrutiny over security concerns, and recently published studies by MIT and Trail of Bits uncovered significant security weaknesses in the app.

While security experts long have dismissed mobile voting as inherently risky, proponents of mobile-voting have maintained that the apps and process are more secure and private, for example, than the standard practice of sending PDF-based ballots via unencrypted email to military personnel overseas.

Voatz recently had updated its bug bounty policy on HackerOne to say that it could not "guarantee safe harbor" for researchers who discover flaws in its software under the program, CyberScoop said in its report.
April 20, 2020

The Honorable Ellen F. Rosenblum  
Office of the Attorney General  
Commerce Building  
158 12th St. NE  
Salem, OR 97301

Dear Attorney General Rosenblum,

We write to you to urge you to initiate an investigation into the voting system vendor Voatz for advancing potential false claims and deceptive marketing practices while promoting its mobile voting application in Oregon that may violate the Unlawful Trade Practices Act, Or. Rev. Stat. § 646.607; fraudulent misrepresentation; or any other violation of state law.¹

Voatz is Boston-based startup company that is developing and aggressively marketing an internet-based voting system that enables voters to cast a ballot from application loaded on to their mobile phones. In 2019, Jackson and Umatilla counties contracted to have Voatz offer its internet voting system to voters eligible under the Uniformed and Overseas Citizen Absentee Voting Act (UOCAVA) for Oregon’s 2020 general elections.

Voatz’s campaign to promote its voting system in Oregon has included bogus claims of “military grade security,”² public statements asserting that votes cast on its platform could not be deleted or altered,³ and published materials⁴ and presentations⁵ promising that Voatz’s system was robustly vetted and secure.⁶ Though many computer security