Stat 2, Summer 2010: Notation review

This is a brief review of some notation that you might come across in this course. You have already encountered x, usually called the *independent* (or *explanatory* or *predictor*) variable; and y, the *dependent* variable.

When we have two variables, we can compute the average of each set of data by adding up all the values and dividing by the total number of values. For example, suppose we had two lists of data, and we call one x, and the other y :

x	y
5	2
-1	4
0	3
3	5

We denote the *average* of x by $\bar{x}$, where

$$\bar{x} = \frac{5 + (-1) + 0 + 3}{4} = \frac{7}{4} = 1.75.$$

Similarly, the average of the y-data is denoted $\bar{y}$ and is given by

$$\bar{y} = \frac{2 + 4 + 3 + 5}{4} = 3.5.$$

Now, this was easy to write, since each list had only four values. But if it has many, to denote the sum of the list, we use the following shorthand:

$$\sum_{i=1}^{4} x_i$$

Now in the expression above, i refers to the *index*. In this example,

$$x_1 = 5,\ x_2 = -1,\ x_3 = 0,\ x_4 = 3$$

So, i begins at 1 and goes until 4. Sometimes we only want to add some of the x-values, say we only want to add the last three values, then we would have :

$$\sum_{i=2}^{4} x_i = x_2 + x_3 + x_4 = \ -1 + 0 + 3$$

In this way, we can use $\sum$ to denote any sum that we are considering:

$$\bar{x} = \sum_{i=1}^{4} x_i = \frac{x_1 + x_2 + x_3 + x_4}{4}$$

and

$$\bar{y} = \sum_{i=1}^{4} y_i = \frac{y_1 + y_2 + y_3 + y_4}{4}$$

Sometimes, we might want to sum a *function* of the x-values, such as the squares of the values:

$$\sum_{i=1}^{n} x_i^2$$

In our example,

$$\begin{aligned}
\sum_{i=1}^{4} x_i^2 &= x_1^2 + x_2^2 + x_3^2 + x_4^2 \\
&= 5^2 + (-1)^2 + 0^2 + 3^2 \\
&= 25 + 1 + 0 + 9 \\
&= 35
\end{aligned}$$

And $SD(x)$, the average deviation from the mean, can be written as :

$$\begin{aligned}
SD(x) &= \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}} \\
&= \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2 + (x_4 - \bar{x})^2}{4}} \\
&= \sqrt{\frac{(5 - 1.75)^2 + (-1 - 1.75)^2 + (0 - 1.75)^2 + (3 - 1.75)^2}{4}} \\
&\approx 5.7.
\end{aligned}$$