1. The game is symmetric, therefore $A = B^T$. Now let 

$$
\tilde{K} = \{(x, x) : x \in \Delta_m\} = \Delta_m \times \Delta_m.
$$

Note that $\tilde{K}$ is closed, bounded and convex. We need to define a map $f : \tilde{K} \to \tilde{K}$ such that $f$ is continuous, and then we can apply Brouwer's fixed point theorem. (And then, following the proof in the book, we need to show that this fixed point must be a Nash Equilibrium.) Define $c_i$ as in the notes, and note that $d_i = c_i$ because of symmetry.

$$
c_i = c_i(x, x) = \max\{A_{(i)}x - x^T Ax, 0\}
$$

Note that since the game is symmetric, $A_{(i)}x = x^T B^{(i)}$, and $c_i$ gives the gain (if any) of either player by switching from strategy $x$ to pure strategy $e_i$.

Now we can define our function $f$ by $f(x, x) = (y, y)$ where

$$
y_i = \frac{x_i + c_i}{1 + \sum_{i=1}^m c_i}
$$

We see that $y_i$ is clearly non-negative, and $\sum_{i=1}^m y_i = 1$, therefore $y_i \in \Delta_m$. Also $f$ is continuous, since $c_i$ is continuous.

By Brouwer’s fixed point theorem, there exists a fixed point for $f$, say $p$, with $f(p, p) = (p, p)$. We need to show that $p$ must be a Nash equilibrium.

Since $p_i = \frac{p_i + c_i}{1 + \sum_{i=1}^m c_i}$, $\implies p_i \sum_{i=1}^m c_i = c_i$. This gives us that $c_i(p, p) = 0$ for each $i$. Therefore, $A_{(i)}p \leq pAp$ for each $i$, which implies that for every $x \in \Delta_m$,

$$
$$

Since $A_{(i)}p = p^T B^{(i)}$, we see that $p^T B^{(i)} x \leq pAp$ as well. \hfill \Box

2. Let the drivers be $x_1, \ldots, x_6$ and their associated costs be $c_1, \ldots, c_6$. Then it is clear that $c_1 = 19$, since $k$ will increment by 1. Now $x_2$ will choose to use the other possible route to $D$, and thus $c_2$ will also be 19. Proceeding in this way for each driver, we see that $c_3 = \min(25, 25) = 25$, $c_4 = 25$, $c_5 = c_6 = 31$, bringing the total cost to 150 units.

If a super highway is introduced along segment $AC$, then drivers 1, 2, 3 and 5 will go on this to reduce their cost, and drivers 4 and 6 will go along the segments $AB - BD$ to minimize their costs. This will bring the total cost to 102 units.

3. Let the pure strategies for player I be given by $s_1$ and $s_2$ where $s_1$ is the route $AD - DC$ and $s_2$ is the route $AB - BC$, and the pure strategies for player II be given by $r_1$ and $r_2$ where $r_1$ is the route $BC - CD$ and $r_2$ is the route $BA - AD$. This results in the following payoff matrix: (with payoff = - cost).

$$
\begin{pmatrix}
    s_1 & s_2 \\
    s_1 & (-5, -5) & (-7, -8) \\
    s_2 & (-5, -4) & (-7, -7)
\end{pmatrix}
$$

The pure Nash equilibria are at $(s_1, r_1)$ and $(s_2, r_1)$.