Stat 155 Fall 2009: Solutions to Homework 3

(was due October 1, 2009)

1. The payoff matrix is shown below, with player I's choices along the rows, and player II's choices along the columns.

$$\begin{array}{cc} & \begin{array}{cc} \text{Black} & \text{Red} \end{array} \\ \begin{array}{c} \text{Black} \\ \text{Red} \end{array} & \begin{pmatrix} 1 & -2 \\ -7 & 8 \end{pmatrix} \end{array}$$

   Using equalizing strategies, as discussed in class, we will solve for the optimal p to find the value of the game where p is the probability that Player I chooses the Black Ace, and $1-p$ is the probability that Player I chooses the Red 8. Equating Player I's expected payoffs under the two cases of Player II playing Black or Red, we get:

$$1 \cdot p - 7(1-p) = -2p + 8(1-p)$$

   We solve this to get $p = 5/6$. This gives the value of the game to be $-1/3$. If Player II's optimal strategy is $(q, 1-q)$, where q is the probability that Player II chooses Black, then we can use the same method to solve for q, getting $q = 5/9$. Thus,
   Player I's optimal strategy: $(5/6, 1/6)$,
   Player II's optimal strategy: $(5/9, 4/9)$, and
   Value of the game $= -1/3$.

2. Payoff matrix:

$$\begin{array}{cc} & \begin{array}{ccc} \text{Rock} & \text{Paper} & \text{Scissors} \end{array} \\ \begin{array}{c} \text{Rock} \\ \text{Paper} \\ \text{Scissors} \end{array} & \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \end{array}$$

   Let player I's strategy be (p_1, p_2, p_3), where $p_1 + p_2 + p_3 = 1$. Then we can set up the equations to get:
   $p_2 - p_3 = p_3 - p_1 = p_1 - p_2$, and using the constraint on the p_i, this is easily solved to give us the optimal strategy $(1/3, 1/3, 1/3)$ for player I (and player II, as a matter of fact).

3. First, let us set up the payoff matrix for the original game (without Olaf's suggested side payment). Alex's choices are along the rows and Olaf's along the columns:

$$\begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} 55 & -10 \\ -10 & 110 \end{pmatrix} \end{array}$$

   Solving for the optimal strategies, we get that they should both use the same strategies of $(24/27, 13/37)$, and the value of the game works out to be about $+32.16$ cents, thus favoring Alex. If he pays Olaf 42 cents before each game, the game's value changes to -9.84 cents, and thus will favor Olaf. It will still be an unfair game.