Stat 155 Fall 2009: Solutions to Homework 2

(was due September 17, 2009)

1. The Sprague-Grundy function of the 2×3 rectangular piece of chocolate for the game of Chomp is enumerated below. It was obtained using the graph on page 11 of the text, starting with assigning a value of 0 to the terminal position, and then using the definition of g.

   $g(\circ) = g\left(\begin{smallmatrix}\bullet & \\ \circ & \bullet\end{smallmatrix}\right) = g\left(\begin{smallmatrix}\bullet & \bullet & \\ \circ & \bullet & \bullet\end{smallmatrix}\right) = 0$

   $g\left(\begin{smallmatrix}\bullet \\ \circ\end{smallmatrix}\right) = g\left(\begin{smallmatrix}\circ & \bullet\end{smallmatrix}\right) = 1$

   $g\left(\begin{smallmatrix}\circ & \bullet & \bullet\end{smallmatrix}\right) = g\left(\begin{smallmatrix}\bullet & \bullet \\ \circ & \bullet\end{smallmatrix}\right) = 2$

   $g\left(\begin{smallmatrix}\bullet & & \\ \circ & \bullet & \bullet\end{smallmatrix}\right) = 3$

   $g\left(\begin{smallmatrix}\bullet & \bullet & \bullet \\ \circ & \bullet & \bullet\end{smallmatrix}\right) = 4$

2. The P-positions of G_1 are given by the positions x where $x \equiv 0 \bmod 7$ or $x \equiv 2 \bmod 7$. Thus, $g_1(100) = 0$.
   Writing out the Sprague-Grundy values for G_2, we see that they are periodic, with period 8.
   $g_2(x) = 0$, for $x \equiv 0, 1 \bmod 8$, $g_2(x) = 1$, for $x \equiv 2, 3 \bmod 8$, $g_2(x) = 2$, for $x \equiv 4, 5 \bmod 8$, and $g_2(x) = 3$, for $x \equiv 6, 7 \bmod 8$.
   Since $100 \equiv 4 \bmod 8$, we have $g_2(100) = 2$.
   Finally, $g_3(100) = 100 \bmod 21 = 16$. Then, using the sum theorem,

   $$g(100, 100, 100) = g_1(100) \oplus g_2(100) \oplus g_3(100) = 0 \oplus 2 \oplus 16 = 18.$$

   Since the Sprague-Grundy value of the position is non-zero, it must be an N-position.

3. Note that $(1, 2, 3)$ denotes 3 piles of *"Lasker"* heaps, not nim-heaps. By the sum theorem, $g(1, 2, 3) = g_1(1) \oplus g_2(2) \oplus g_3(3)$, where g_1, g_2, g_3 denote the Sprague-Grundy functions of the single pile Lasker's Nim games. Using the definition of the Sprague-Grundy function, the values of the g_i can be worked out. The only one different from regular nim is 3. Since, in addition to the usual nim moves, 3 can be split into the two smaller piles of sizes 2 and 1, we find that the Sprague-Grundy value of (3) is in fact 4. Thus we have $g(1, 2, 3) = 1 \oplus 2 \oplus 4 = 7$ and $(1, 2, 3)$ is an N-position for Lasker's Nim.

4. The winning move is to split the pile with 3 chips into two smaller piles with 2 chips and 1 chip. This gives us the position $(1, 2, 1, 2)$ which has Sprague-Grundy value 0.