Stat 155 Fall 2009: Practice Final, December 7, 2009
Duration: 3 hours

Please show **all** steps.

1. Solve the game of nim in which you are only allowed to take an **odd** number of beans from a heap. Is the position (5,45,3) in N or in P? If it is in N, what would be a winning first move?

2. Consider the following coin-turning game: there are a finite number of coins, each showing heads or tails. A move consists of turning over coins. Any number of coins may be turned over, but they must be consecutive, and the rightmost coin turned over must go from heads to tails. Find the Sprague-Grundy function for this game.

3. Player II chooses a number $j \in \{1, 2, 3, 4\}$ and I tries to guess what it is. If the guess is correct, I wins a dollar. If the guess is too high, I loses a dollar, and if it is too low, there is no payoff. Set up the matrix of this game, and solve it.

4. Find the optimal strategies for, and the value of the game whose matrix is given by:

$$\begin{pmatrix} 3 & -7 & 0 & -4 \\ -6 & 1 & -1 & 5 \end{pmatrix}$$

5. State and prove Nash's theorem (you may assume Brouwer's fixed point theoerem).

6. Find all Nash equilibria in the general sum game given by:

$$\begin{pmatrix} (3,3) & (0,2) \\ (2,1) & (5,5) \end{pmatrix}$$

7. State and prove Arrow's theorem.

8. Find the Shapley-Shubik index of the game that consists of 5 voters: A,B,C,D,E, who vote by majority rule, but A has a veto.

9. Show that the Gale-Shapley algorithm for finding a stable matching yields a stable matching.

10. Consider the game with 5 sellers, where L_1, L_2, L_3 each have one lefthand glove, and R_1 and R_2 each have one righthand glove. The value of a coalition is the number of pairs of gloves it has. Find the core of this game, and the Shapley value. Is the Shapley value in the core?