Stat 155 Fall 2009: Midterm guide

- The exam will cover **combinatorial games**, impartial and partisan; and 2-person zero-sum games (Chapters 2,3 in *Game Theory, Alive*, and Parts I, II of Ferguson. We did not do everything in these chapters from either of these books, so please also use your lecture notes as a guide.)

- **Definitions:** You must know the definitions that we have covered in class, be able to compute nim-sums, Sprague-Grundy values, so that you can decide which player has a winning strategy, use the colon principle and the fusion principle (and state them, if you have to). You should be able to find the Sprague-Grundy function of a game, and of the graph of the game.

- **Theorems:** Strategy-stealing arguments (hex and chomp), various induction arguments, Bouton's theorem, Sprague-Grundy theorem, Sum theorem, proofs that a given graph is a tree, and various properties that characterize trees. These are the main theorems, but you should be pretty comfortable with the kinds of arguments that we went over in class.

- From Chapter 3, the main theorem is the Minimax theorem. You should be able to state and prove the supporting hyperplane theorem too. What are pure strategies, mixed strategies, optimal strategies, saddle points, dominating strategies, payoff vectors and matrices. What is the value of the game. How do optimal strategies relate to the value of the game etc. You should be able to solve a small game, given the payoff matrix.

- Some practice problems:

  1. page 44 (GTA) 2.9 , 2.10, 2.14, 2.15
  2. Recall Nimble, where coins were placed on a strip of squares, and you realised that it was Nim in disguise. Here is a modification of Nimble: Place the coins on the squares, but now you can have at most one coin on each square, and you may not jump over another coin. A move is to slide a coin as far left as you like, up to, but not onto, or over, the next coin, not off the end of the strip.
  3. Recall Grundy's game, discussed in class, which is played with heaps of beans, and a move is to split the heap into two unequal heaps. Analyse a modification of this game, where splitting heaps into two *equal* heaps is allowed too. The game is over when all heaps are of size 1.
  4. Find the Sprague-Grundy function for the subtraction game with subtraction set $\{2, 4, 7\}$.
  5. Problem 4 (Turning Turtles) on page I-12 in Ferguson's text.
  6. Problem 5 on page I-26 in Ferguson's text.
  7. Problem 3.5,page 70 in GTA
  8. Problem 3.10, page 72 in GTA
  9. Problems 4,5,6 on page II-14 in Ferguson's text.