Homework 1

1. Give two examples of a 3×3 projection matrices that project onto one and two dimensional subspaces (thus four examples in all). Identify the subspaces.

2. Let $\mathbf{X}$ be a random n-vector and let $\mathbf{Y}$ be a random vector with $Y_1 = X_1$, $Y_i = X_i - X_{i-1}$, $i = 1, 2, \ldots, n$. Use matrix methods to solve the following:

   (a) If the X_i are independent random variables, find the covariance matrix of $\mathbf{Y}$.

   (b) If the Y_i are independent random variables, find the covariance matrix of $\mathbf{X}$.

3. Let $X_1, X_2, \ldots, X_n$ be independent random variables with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2$. Let

$$Q = \sum_{i=1}^{n-1} (X_{i+1} - X_i)^2$$

   Use matrix methods to find $E(Q)$ and show that $Q/2(n-1)$ is an unbiased estimate of σ^2.