# Chance Models for Measurement Error

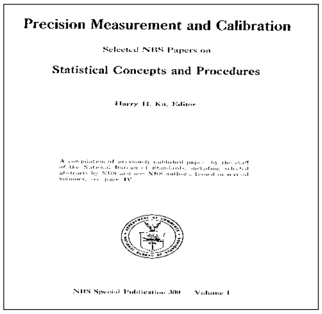


## Chance models are also called "stochastic" models

**sto.chas.tic** adj [Gk stochastikos skillful in aiming, fr. stochazesthai to aim at, guess at, fr. stochos target, aim, guess] 1: random; specif: involving a random variable <a ~ process> 2: involving chance or probability: probabilistic. sto.chas.ti.cal.ly adv



## Repeated Measurements of the Same Object Are Subject to Error



**Precision Measurement and Calibration**

Selected NBS Papers on

**Statistical Concepts and Procedures**

Harry H. Ku, Editor

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## Mass Measurement Requirements

One role of the Bureau is to provide an extension of the mass measurement unit into the facilities of those who must use mass values to do other useful work. . . .

Figure 1

These large weights, for example, are for use by another part of the Bureau to calibrate force measuring devices.

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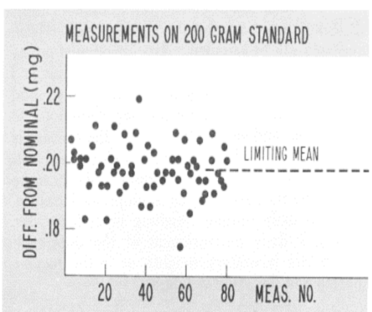


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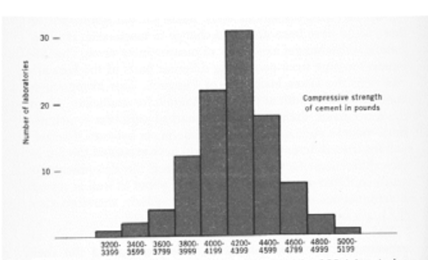


Histogram for cement tests reported by 102 laboratories.

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Figure 4. Histogram of 100 analyses for magnesium

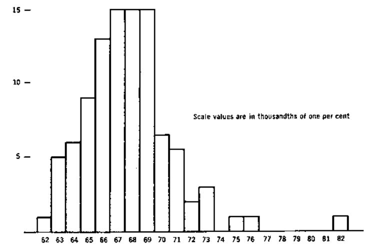
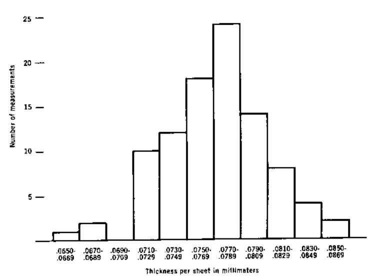





Figure 2. Histogram for 95 measurements of paper thickness






It's "natural" to average the measurements, thinking that the average should be more accurate than a single measurement.

Why would this be so? If it is, how much more accurate?

To address these questions, we need a model.

## Model for Measurement Error

measurement = exact value + bias + error

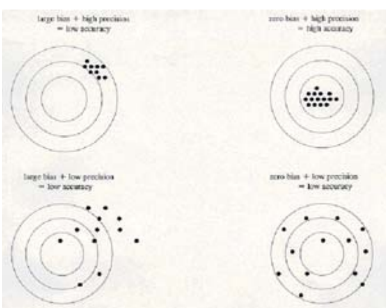
= exact value + error (if no bias)

The errors are "random" and "independent." They are like draws with replacement from a box with many, many tickets. Box has an average 0 and some unknown SD. We pretend that the errors are a "sample" from this box.

The book calls this the "Gauss model."



## Precision, Bias, and Accuracy






With this model, we can use our previous results:

EV of sample mean = exact value + bias

SE of sample mean = Box SD $/\sqrt{\text{sample size}}$

We can estimate the box SD by the SD of the values in the sample. That will give us the SE of the sample mean and we can form confidence intervals if we wish.

# Aside: Common Notation

1st measurement

True value. Population mean

$X_1 = \mu + e_1$

$X_2 = \mu + e_2$

.

.

.

$X_n = \mu + e_n$

Independent random errors

SD of errors = $\sigma$

$\bar{X}$ = sample average



# With this notation:

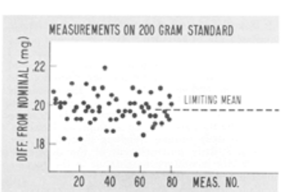
EV of sample average = $E(\bar{X}) = \mu$

SE of $\bar{X}$ = $\dfrac{\sigma}{\sqrt{n}}$

The box SD, $\sigma$, is estimated by the sample SD, which is denoted by **s**



# Example




sample mean =

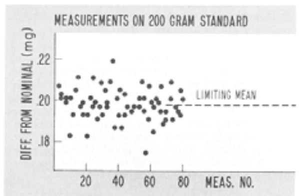
sample SD =

sample size = 80

SE of sample mean =

95% confidence interval =

An important part of the model: the errors are "independent." There is no pattern to them. One error cannot be predicted from another, as in draws from a box with replacement.





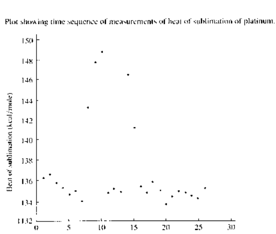

# Example

| Heats of Sublimation of Platinum (kcal/mol) | | | | | | | |
|---|---|---|---|---|---|---|---|
| 136.3 | 136.6 | 135.8 | 135.4 | 134.7 | 135.0 | 134.1 | 143.3 |
| 147.8 | 148.8 | 134.8 | 135.2 | 134.9 | 146.5 | 141.2 | 135.4 |
| 134.8 | 135.8 | 135.0 | 133.7 | 134.4 | 134.9 | 134.8 | 134.5 |
| 134.3 | 135.2 | | | | | | |

average = 137.05



Is independence a good model?




mean = 137.05

median = 135.1

## A Variation on the Measurement Error Model: Counting Frequencies

What is the chance that a thumbtack lands point up?

What's the chance that a shimbui lands flat side up?



Box model: the box has a certain percentage of tickets =1 and the rest = 0. We want to know that percent by drawing from the box. We model the trials as draws with replacement from such a box.

EV of sample percent = percentage in box

SE of sample percent =

$$100 \times \sqrt{(\text{fraction 1's}) \times (\text{fraction 0's})} \div \sqrt{\text{sample size}}$$



## More Common Notation

box fraction = p

sample fraction = $\hat{p}$

SE of $\hat{p}$ = $\dfrac{\sqrt{p \times (1-p)}}{\sqrt{n}}$

? Measurements on the 200 gram standard had an SD = .01. How many measurements would have to be taken so that the SE of the average equaled .001?

SE of average = .001 = SD / $\sqrt{number}$

.01

$\sqrt{number}$ = .001/.01 = 10

So, 100 measurements are needed.



? How many times would I have to toss the shimbui to have the error in my estimate of the chance of it landing flat side up less than 1%?



? An airplane has 100 passengers. From previous experience, the airline thinks that the average baggage weight will be 50 pounds with an SD of about 15 pounds. The airline can expect that the total baggage weight on this flight will be _____ plus or minus _____ or so?

EV of sum =

SE of sum =

EV of sum = pounds. SE = pounds.

The chance that the total weight is less than _____ is about 99%.

Use the normal curve with EV and SE as above.