Statistics 135: Fall 2004
Final Exam

Name:
SID#:

There are 10 problems and the number of points for each is shown in parentheses. There is a normal table at the end. *Show your work.*

1. The designer of a sample survey stratifies a population into two strata, H and L. H contains 100,000 people and L contains 500,000. He decides to allocate 100 samples to stratum H and 200 to stratum L, taking a simple random sample in each stratum.

   (a) (5) How should he estimate the population mean?

   (b) (5) Suppose that the population standard deviation in stratum H is 20 and the standard deviation in stratum L is 10. What will be the standard error of his estimate?

   (c) (5) Would it be better to use proportional allocation?

2. George spins a coin three times and observes no heads. He then gives the coin to Hilary. She spins it until the first head occurs, and ends up spinning it four times in all. Let θ denote the probability the coin comes up heads.

   (a) (5) What is the likelihood of θ?

   (b) (5) What is the MLE of θ?

3. Suppose that a random variable X has the probability density $f(x) = (\theta + 1)x^{\theta}$, $0 \leq x \leq 1$. Suppose that $X_1, X_2, \ldots, X_n$ are iid with this density.

   (a) (5) What is the maximum likelihood estimate of θ?

   (b) (5) What is the approximate variance of the MLE?

4. The intensity of light reflected by an object is measured. Suppose there are two types of possible objects, A and B. If the object is of type A, the measurement is normally distributed with mean 100 and standard deviation 25; if it is of type B, the measurement is normally distributed with mean 125 and standard deviation 25. A single measurement is taken with the value $X = 120$.

   (a) (5) What is the likelihood ratio?

   (b) (5) If the prior probabilities of A and B are equal (1/2 each), what is the posterior probability that the item is of type B?

   (c) (5) Suppose that a decision rule has been formulated that declares the object to be of type B if $X > 125$. What is the significance level associated with this rule?

   (d) (5) What is the power of this test?

   (e) (5) What is the p-value when $X = 120$?

5. 50 rats were randomly divided into two groups of 25 each. The rats in one group were given steroids. They were then timed running a maze. The rats in the non-steroid group had an average time of 10 seconds with an SD equal to 2 seconds and the rats in the steroid group had an average time of 9 seconds with an SD equal to 5 seconds.

   (a) (5) What is the SE of the difference of the two averages?

   (b) (5) Use the information given to calculate a test statistic for testing the null hypothesis that steroids made no difference.

   (c) (5) What is the P-value of the test?

6. (15) Three objects are located on a line at points $p_1 < p_2 < p_3$. These locations are not precisely known. A surveyor makes the following measurements:

   (a) He stands at the origin and measures the three distances from there to p_1, p_2 and p_3. Let these measurements be denoted by Y_1, Y_2, Y_3.

   (b) He goes to p_1 and measures the distances from there to p_2 and p_3. Let these measurements be denoted by Y_4, Y_5.

   (c) He goes to p_2 and measures the distance from there to p_3. Denote this measurement by Y_6.

   He thus makes six measurements in all, and they are all subject to error. In order to estimate the values p_1, p_2, p_3, he decides to combine all the measurements by the method of least squares. Using matrix notation, explain clearly how the least squares estimates would be calculated (you don't have to do the actual calculations).

7. Consider the standard linear model $Y = X\beta + \epsilon$, where Y is a n-vector and β is a p-vector. As usual assume that the errors are uncorrelated with constant variance σ^2. The least squares estimate of β is $\hat{\beta} = (X^T X)^{-1} X^T Y$.

   (a) (5) What are the fitted values $\hat{Y}$? Derive their covariance matrix.

   (b) (5) Show that $\sum_{i=1}^{n} Var(\hat{Y}_i) = p\sigma^2$.

8. (5) Let Z be a random vector with 4 components and covariance matrix $\sigma^2 I$. Let $U = Z_1 + Z_2 + Z_3 + Z_4$ and $V = (Z_1 + Z_2) - (Z_3 + Z_4)$. Use matrix methods to find $Cov(U, V)$.

9. (5) Do women have different patterns of work behavior than men? A random sample of graduates of a business school was polled and classified by gender and workaholism type, resulting in the following table:

Work Type	Female	Male
Work Enthusiasts	20	41
Workaholics	32	37
Unengaged Workers	43	52
Relaxed Workers	24	27
Disenchanted Workers	37	30

How would you test whether there was a gender difference? You do not have to calculate the value of the test statistic, but explain clearly and unambiguously what the alternative and null hypotheses are, how the test statistic would be calculated, and how you would assess statistical significance.

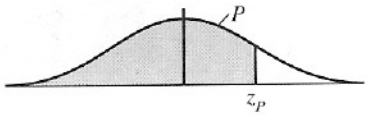
10. (5) For each of 50 subjects an investigator measures IQ on a standard test and reaction time to a physical stimulus. Denote the measurements from subject i as x_i, y_i, for $i = 1, 2, \ldots n$. She then calculates the correlation coefficient,

$$r = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - \bar{y})^2}}$$

Explain clearly and unambiguously how the standard error of r could be approximately found by a bootstrap technique.

TABLE 2

Cumulative Normal Distribution—Values of P Corresponding to z_p for the Normal Curve



z is the standard normal variable. The value of P for $-z_p$ equals 1 minus the value of P for $+z_p$; for example, the P for -1.62 equals $1 - .9474 = .0526$.

z_p	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998