

550 Chapter 14 Linear Least Squares

normally distributed. The normality assumption, or its approximation, makes possible the construction of confidence intervals and hypothesis tests. It can then be shown that

$$\frac{\hat{\beta}_i - \beta_i}{s_{\hat{\beta}_i}} \sim t_{n-2}$$

which allows the t distribution to be used for confidence intervals and hypothesis tests.

EXAMPLE A We apply these procedures to the 21 data points on chromatographic peak area. The following table presents some of the statistics from the fit (tables like this are produced by regression programs of software packages):

Coefficient	Estimate	Standard Error	t Value
β_0	.0729	.0297	2.45
β_1	10.77	.27	40.20

The estimated standard deviation of the errors is $s = .068$. The standard error of the intercept is $s_{\hat{\beta}_0} = .0297$. A 95% confidence interval for the intercept, β_0 , based on the t distribution with 19 df is

$$\hat{\beta}_0 \pm t_{19}(.025)s_{\hat{\beta}_0}$$

or (.011, .135). Similarly, a 95% confidence interval for the slope, β_1 , is

$$\hat{\beta}_1 \pm t_{19}(.025)s_{\hat{\beta}_1}$$

or (10.21, 11.33). To test the null hypothesis $H_0: \beta_0 = 0$, we would use the t statistic $\hat{\beta}_0/s_{\hat{\beta}_0} = 2.45$. The hypothesis would be rejected at significance level $\alpha = .05$, so there is strong evidence that the intercept is nonzero. ■

14.2.2 Assessing the Fit

As an aid in assessing the quality of the fit, we will make extensive use of the residuals, which are the differences between the observed and fitted values:

$$\hat{e}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i$$

It is most useful to examine the residuals graphically. Plots of the residuals versus the x values may reveal systematic misfit or ways in which the data do not conform to the fitted model. Ideally, the residuals should show no relation to the x values, and the plot should look like a horizontal blur.

EXAMPLE A Figure 14.5 is a plot of the residuals for the data on chromatographic peak area. There is no apparent deviation from randomness in the residuals, so this plot confirms