We have

$$R = g(I) = \frac{V_0}{I}$$

$$g'(\mu_I) = -\frac{V_0}{\mu_I^2} \quad g''(\mu_I) = \frac{2V_0}{\mu_I^3}$$

Thus,

$$\mu_R \approx \frac{V_0}{\mu_I} + \frac{V_0}{\mu_I^3}\sigma_I^2$$

$$\sigma_R^2 \approx \frac{V_0^2}{\mu_I^4}\sigma_I^2$$

We see that the variability of R depends on both the mean level of I and the variance of I. This makes sense, since if I is quite small, small variations in I will result in large variations in $R = V_0/I$, whereas if I is large, small variations will not affect R as much. The second-order correction factor for μ_R also depends on μ_I and is large if μ_I is small. In fact, when I is near zero, the function $g(I) = V_0/I$ is quite nonlinear, and the linearization is not a good approximation. ■

EXAMPLE **B** This example examines the accuracy of the approximations using a simple test case. We choose the function $g(x) = \sqrt{x}$ and consider two cases: X uniform on [0, 1], and X uniform on [1, 2]. The graph of $g(x)$ in Figure 4.9 shows that g is more nearly linear in the latter case, so we would expect the approximations to work better there.

Let $Y = \sqrt{X}$; because X is uniform on [0, 1],

$$E(Y) = \int_0^1 \sqrt{x}\, dx = \tfrac{2}{3}$$

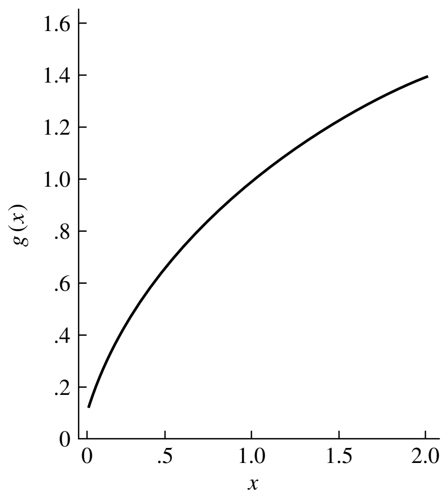

FIGURE **4.9** The function $g(x) = \sqrt{x}$ is more nearly linear over the interval [1, 2] than over the interval [0, 1].