THEOREM A

Suppose that $U = a + \sum_{i=1}^{n} b_i X_i$ and $V = c + \sum_{j=1}^{m} d_j Y_j$. Then

$$\mathrm{Cov}(U, V) = \sum_{i=1}^{n} \sum_{j=1}^{m} b_i d_j \mathrm{Cov}(X_i, Y_j)$$ ■

This theorem has many applications. In particular, since $\mathrm{Var}(X) = \mathrm{Cov}(X, X)$,

$$\begin{aligned}\mathrm{Var}(X+Y) &= \mathrm{Cov}(X+Y, X+Y)\\ &= \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\mathrm{Cov}(X, Y)\end{aligned}$$

More generally, we have the following result for the variance of a linear combination of random variables.

COROLLARY A

$\mathrm{Var}(a + \sum_{i=1}^{n} b_i X_i) = \sum_{i=1}^{n} \sum_{j=1}^{n} b_i b_j \mathrm{Cov}(X_i, X_j)$. ■

If the X_i are independent, then $\mathrm{Cov}(X_i, X_j) = 0$ for $i \neq j$, and we have another corollary.

COROLLARY B

$\mathrm{Var}(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} \mathrm{Var}(X_i)$, if the X_i are independent. ■

Corollary B is very useful. Note that $E(\sum X_i) = \sum E(X_i)$ whether or not the X_i are independent, but it is generally *not* the case that $\mathrm{Var}(\sum X_i) = \sum \mathrm{Var}(X_i)$.

EXAMPLE B Finding the variance of a binomial random variable from the definition of variance and the frequency function of the binomial distribution is not easy (try it). But expressing a binomial random variable as a sum of independent Bernoulli random variables makes the computation of the variance trivial. Specifically, if Y is a binomial random variable, it can be expressed as $Y = X_1 + X_2 + \cdots + X_n$, where the X_i are independent Bernoulli random variables with $P(X_i = 1) = p$. We saw earlier (Example A in Section 4.2) that $\mathrm{Var}(X_i) = p(1-p)$, from which it follows from Corollary B that $\mathrm{Var}(Y) = np(1-p)$. ■

EXAMPLE C *Random Walk*

A drunken walker starts out at a point x_0 on the real line. He ta[illegible] step o[illegible]th X_1, which is a random variable with expected value μ and varia[illegible], and his position