47. a. $\hat{\theta} = \overline{X}/(\overline{X} - x_0)$
b. $\tilde{\theta} = n/(\Sigma \log X_i - n \log x_0)$
c. $\text{Var}(\tilde{\theta}) \approx \theta^2/n$

49. a. Let $\hat{p}$ be the proportion of the n events that go forward. Then $\hat{\alpha} = 4\hat{p} - 2$.
b. $\text{Var}(\hat{\alpha}) = (2 - \alpha)(2 + \alpha)/n$

53. a. $\hat{\theta} = 2\overline{X}$; $E(\hat{\theta}) = \theta$; $\text{Var}(\hat{\theta}) = \theta^2/3n$
b. $\tilde{\theta} = \max(X_1, X_2, \ldots, X_n)$
c. $E(\tilde{\theta}) = n\theta/(n+1)$; bias $= -\theta/(n+1)$; $\text{Var}(\hat{\theta}) = n\theta^2/(n+2)(n+1)^2$; MSE $= 2\theta^2/(n+1)(n+2)$
d. $\theta^* = (n+1)\tilde{\theta}/n$

55. a. Let n_1, n_2, n_3, n_4 denote the counts. The mle of θ is the positive root of the equation

$$(n_1 + n_2 + n_3 + n_4)\theta^2 - (n_1 - 2n_2 - 2n_3 - n_4)\theta - 2n_4 = 0$$

The asymptotic variance is $\text{Var}(\hat{\theta}) = 2(2+\theta)(1-\theta)\theta/(n_1 + n_2 + n_3 + n_4)(1+\theta)$. For these data, $\hat{\theta} = .0357$ and $s_{\hat{\theta}} = .0057$.
b. An approximate 95% confidence interval is $.0357 \pm .0112$.

57. a. s^2 is unbiased. **b.** $\hat{\sigma}^2$ has smaller MSE. **c.** $\rho = 1/(n+1)$

59. b. $\hat{\alpha} = (n_1 + n_2 - n_3)/(n_1 + n_2 + n_3)$ if this quantity is positive and 0 otherwise.

63. In case (1) the posterior is $\beta(4, 98)$ and the posterior mean is 0.039. In case (2) the posterior is $\beta(3.5, 102)$ and the posterior mean is 0.033. The posterior for case (2) rises more steeply and falls off more rapidly than that of case (1).

65. $\mu_0 = 16.25$, $\xi_0 = 80$

71. $\prod_{i=1}^{n} (1 + X_i)$

73. $\sum_{i=1}^{n} X_i^2$

Chapter 9

1. a. $\alpha = .002$ **b.** power $= .349$

3. a. $\alpha = .046$

5. F, F, F, F, F, F, F, T

7. Reject when $\sum X_i > c$. Since under H_0, $\sum X_i$ follows a Poisson distribution with parameter $n\lambda$, c can be chosen so that $P(\sum X_i > c | H_0) = \alpha$.

9. For $\alpha = .10$, the test rejects for $\overline{X} > 2.56$, and the power is .2981. For $\alpha = .01$, the test rejects for $\overline{X} > 4.66$, and the power is .0571.

17. a. $LR = \frac{\sigma_1}{\sigma_0} \exp\left[\frac{1}{2}x^2\left(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_0^2}\right)\right]$. A level α test rejects for $X^2 > \sigma_0^2 \chi_1^2(\alpha)$.
b. Reject for $\sum_{i=1}^{n} X_i^2 > \sigma_0^2 \chi_n^2(\alpha)$ **c.** Yes

19. a. $X < 2/3$ **b.** Reject for large values of X
c. Reject for $X > \sqrt{1 - \alpha}$ **d.** $1 - (1 - \alpha)^3/2$