Name:

Statistics 200B
1996 Midterm Exam

1. Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. geometric random variables; $P(X = k) = p(1-p)^{k-1}$, $k = 1, 2, \ldots$. $E(X_i) = 1/p$.

   (a) What is the maximum likelihood estimate of p?

   (b) What is the approximate large sample variance of the estimate?

   (c) Derive a generalized likelihood ratio test of the hypothesis $p = .5$ versus the alternative $p \neq .5$. What is the form of the rejection region for the test?

   (d) What is the large sample null distribution of $-2 \log \Lambda$?

2. A population of size N contains a subpopulation of size N_1. For concreteness, consider the case in which the population consists of accounts, and the subpopulation consists of a particular kind of account (for example, small business accounts). A simple random sample of size n from the entire population records the value of the account for each sample member. The size (N_1) of the subpopulation is not known. Ignore the finite population correction in answering the following questions.

   (a) How can the total value of the accounts of the subpopulation be estimated?

   (b) Is the estimate unbiased?

   (c) How can the standard error of the estimate be estimated?

3. A friend of yours comes to you with the following problem. Her boss has told her that the middle of a batch of numbers is to be calculated as the difference of the upper and lower quartiles divided by 2. (The upper quartile is that number such that 25% of the numbers are larger than it; the lower quartile is that number such that 25% of the numbers are smaller than it.) She told him that this seemed silly to her, but he insisted that it is mandated by an obscure government (EEC) regulation. Basically, he told her to shut up and just do it. So she did it and she got a number.

   (a) When she faxed the number to her boss, he sent back a fax asking for the standard error of the number she had calculated. After some email interchanges, he told her to assume that the original numbers were normally distributed with unknown mean and variance. How would you advise her to compute the standard error?

   (b) She followed your advice but she comes back to you again. Her boss is not able to find any government regulation that allows him to use the normal distribution. He now wants her to find the standard error without the normality assumption, but just assuming that the numbers are i.i.d. from some unknown common distribution. How would you now advise her to proceed?