Name:

Statistics 200B Final Exam
Spring 1996

1. It is well known that the times between events in a homogeneous Poisson process with intensity μ are independent exponential random variables with density function,

$$f(t) = \frac{1}{\mu}\exp(-t/\mu), \quad t \geq 0$$

   It is also know that the number of events in a time interval of length one is a Poisson random variable with parameter μ, and that the numbers of events in disjoint intervals are independent.

   (a) Suppose that n times, $T_1, \ldots, T_n$ between events are observed. What is the maximum likelihood estimate of μ?

   (b) What is the large sample approximation to the variance of the estimate?

   (c) Describe another method of finding an approximation to the variance.

   (d) Now suppose that in addition to the n times above, the counts, $X_1, \ldots, X_m$ in m disjoint intervals each of length one are observed. How can this information be combined with the time intervals to yield an improved estimate of μ?

2. Suppose that $X_1, \ldots, X_n$ are independent Poisson random variables with parameter λ_x and that $Y_1, \ldots, Y_m$ are independent Poisson with parameter λ_y.

   (a) Find the likelihood ratio test of the null hypothesis $H_0 : \lambda_x = \lambda_y$.

   (b) What is the large sample null distribution of the test statistic?

   (c) Describe how a permutation test could be used in this situation.

   (d) Describe a method for estimating the parameter $\rho = \lambda_x/\lambda_y$ and how you could estimate the standard deviation of $\hat{\rho}$.

3. Consider a standard multiple linear regression model.

   (a) Show that if the model contains a constant term (an intercept) then the sum of residuals is zero.

   (b) Show that $\sum Y_i \hat{Y}_i = \sum \hat{Y}_i^2$

   (c) What is the covariance between Y_i and $\hat{\beta}_j$?

4. The weights of two objects are to be estimated from four weighings on a balance. The four weighings produce estimates of (1) the weight of object one, (2) the weight of object two, (3) the sum of their weights, and (4) the difference of their weights. Suppose the measurements are corrupted by independent errors with expected values of zero and variances σ^2.

   (a) Find explicit expressions for the least squares estimates of the two weights and the variances of the estimates.

   (b) Consider an alternative method in which the objects are just weighed r times each by themselves, there thus being a total of $2r$ weighings. How many weighings would be necessary to produce estimates with the same variances as those from the previous method, which used four in all?

5. As you undoubtedly know, there has been a rapid increase in the prison population in California. One stated rationale for this increase is that it prevents crime, since prisoners cannot commit crimes (on the outside). Attempts have been made to quantify this. One technique interviews new prisoners and ask them how many crimes they committed during a previous time period. These numbers are then multiplied by the duration of imprisonment and the results are added to estimate how many crimes are being prevented. Give at least three sources of error in this estimate of crime prevented by imprisonment.