Statistical and software tools for quantifying uncertainty in event attribution

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Outline

• Statistical methods for event attribution
  • Sources of uncertainty
  • Risk ratio estimation methods
  • Methods for quantifying uncertainty
  • Recommendations
• Illustration with Texas heatwave of 2011
  • Sensitivity analysis with respect to event definition
  • Use of climextremes package for risk ratio estimation
• climextremes package
  • Illustration on station precipitation data
  • Features/capabilities
Sources of uncertainty in model-based event attribution

- **Sampling uncertainty**
  - Due to variability in the earth system
  - Amenable to frequentist or Bayesian statistical treatment
  - Uncertainty decreases with larger ensembles

- **Non-sampling uncertainty**
  - Sources:
    - Boundary condition uncertainty
    - Model parametric uncertainty
    - Model structural uncertainty
  - Not amenable to frequentist treatment
  - Does not decrease with larger ensembles
  - Possibly characterized based on sensitivity analysis or drawing from prior distribution over boundary conditions, parameter values, models, etc.
Methods to estimate probabilities for risk ratio (RR)

- Fit a **parametric statistical model** to climate variable values
  - E.g., lognormal or gamma distribution
  - Estimate probability of exceeding the threshold from fitted distribution
  - Strong assumption about appropriateness of model (all data used)
- **Count exceedances of threshold** (binomial sample) amongst climate variable values
  - “Nonparametric” – no distributional assumption
  - More involved to account for dependence (e.g., daily data)
  - High uncertainty when there are very few events, but can be effective for RR
- **Fit extreme value distribution** (e.g., GEV or peaks-over-threshold (POT))
  - Theoretically justified when threshold is far in tail
  - Informed only by extreme values
  - Hard to use with seasonal events because of small sample sizes
Methods to estimate uncertainty in RR

- Methods for **extreme value methods and binomial counting**
  - Asymptotic statistical calculations (delta method / propagation of error)
    - Assumption of normality
  - Bootstrap
    - Standard statistical bootstrap gives a confidence interval, not a Bayesian probability interval
    - Estimation of RR in bootstrap samples often fails (lack of EVA convergence, zeros in binomial counting approach)
  - Likelihood ratio-based interval

- Methods for **binomial counting** (from epidemiology/biostatistics)
  - Wilson’s method
  - Koopman’s method
  - Wang/Shan method

- All but asymptotic and bootstrap can give interval when RR estimate is 0 or infinity, e.g., (12.8, Infinity)
RR confidence intervals (binomial counting) simulation results

Proportion of times interval cannot be calculated (e.g., 0 events / 0 events)
RR confidence intervals (binomial counting) simulation results

Proportion of times lower interval endpoint includes true RR (95% is best)
RR confidence intervals (binomial counting) simulation results

Proportion of times upper interval endpoint includes true RR (95% is best)
RR confidence intervals (binomial counting) simulation results

Average value of lower endpoint (higher is better)

- Wilson
- Koopman
- Wang/Shan
- delta method
- likelihood ratio
- percentile bootstrap
- basic bootstrap
- bootstrap t
- bootstrap BCA

$p_F$

- 0.01
- 0.025
- 0.05
- 0.1
- 0.2
Methods to estimate uncertainty in RR: conclusions

• Bootstrap methods
  • Often fail to provide an interval
  • Poor statistical performance

• Likelihood ratio method
  • Reasonably good statistical performance, but coverage sometimes too low
  • Will work with both extreme value analysis and binomial counting

• Epidemiology/biostatistical methods
  • Work only for binomial counting
  • Koopman and Wang-Shan methods generally perform well

• All methods except Wang-Shan available in climextremes software package
RR analysis example: Texas 2011 heatwave / drought

- CAM5.1 400-member ensembles for factual and counterfactual
- March-August temperature and rainfall over Texas
- Estimation done using climextremes software
- Temperature: 2/0 is RR count-based estimate
- Precipitation: 0/0 is RR count-based estimate

![Graphs showing temperature and precipitation anomalies](image)

(a) temperature anomaly

(b) precipitation anomaly
Texas 2011 temperature analysis

Actual event is 2.62 degree anomaly

<table>
<thead>
<tr>
<th>Event</th>
<th>Number exceedances</th>
<th>RR estimate</th>
<th>EVA lik ratio CI</th>
<th>Binomial lik ratio CI</th>
<th>Binomial Koopman CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.62 (actual)</td>
<td>2/0</td>
<td>Inf</td>
<td>(12.8, Inf)</td>
<td>(1.0, Inf)</td>
<td>(0.7, Inf)</td>
</tr>
<tr>
<td>2.0</td>
<td>43/0</td>
<td>Inf</td>
<td>NA</td>
<td>(31, Inf)</td>
<td>(16, Inf)</td>
</tr>
<tr>
<td>1.5</td>
<td>129/3</td>
<td>43</td>
<td>NA</td>
<td>(19, 133)</td>
<td>(17, 108)</td>
</tr>
<tr>
<td>1.03 (20-year event)</td>
<td>245/11</td>
<td>22</td>
<td>NA</td>
<td>(14, 38)</td>
<td>(14, 36)</td>
</tr>
<tr>
<td>0.73 (10-year event)</td>
<td>314/40</td>
<td>7.9</td>
<td>NA</td>
<td>(6.2, 10.2)</td>
<td>(6.1, 10.1)</td>
</tr>
<tr>
<td>0.43 (5-year event)</td>
<td>357/90</td>
<td>4.0</td>
<td>NA</td>
<td>(3.4, 4.7)</td>
<td>(3.4, 4.6)</td>
</tr>
</tbody>
</table>
Texas 2011 precipitation analysis

- Recall that actual event (40% of historical average precipitation) has no events in factual or counterfactual ensembles
- Extreme value analysis (EVA) gives (0.01, Inf) as interval
- EVA not really appropriate for less extreme events but shown anyway
climextremes software

• High-level goals
  • Operate from Python or R
  • Provide risk ratio calculations and extreme value analysis fitting (GEV and POT)
  • Handle common situations with climate data
  • Designed for both observations and model output

• Technical features
  • Use of covariates for any extreme value distribution parameter (nonstationary fitting)
  • Estimation with uncertainty for risk ratios, return values, return periods, differences in return values
  • Various techniques for estimating uncertainty
  • Statistically rigorous estimation with model ensembles
  • Statistically rigorous treatment of missing values (for POT)
  • Allows weighting (e.g., weighting nearby stations)
climextremes example

- US GHCN Santa Cruz precipitation
  - 1950-2016
  - November-May rainy season
  - 270 missing days
Stationary peaks-over-threshold fit

result = climextremes.fit_pot(numpy.array(exc.y), nBlocks = nyears, threshold = threshold, firstBlock = missing.seasonyear[0], blockIndex = numpy.array(exc.seasonyear), index = numpy.array(exc.day), proportionMissing = numpy.array(missing.propMiss), declustering = 'noruns', returnPeriod = 20, returnValue = 100, bootSE = False)

Nonstationary peaks-over-threshold fit

# linear location trend in time -
# contrast 2015 returnValue and return probability with that for 1950

resultNS = climextremes.fit_pot(numpy.array(exc.y), x = numpy.array(missing.seasonyear), locationFun = 1, nBlocks = nyears, threshold = threshold, firstBlock = missing.seasonyear[0], blockIndex = numpy.array(exc.seasonyear), index = numpy.array(exc.day), proportionMissing = numpy.array(missing.propMiss), declustering = 'noruns', xNew = 2015, xContrast = 1950, returnPeriod = 20, returnValue = 100, bootSE = False)
climextremes example

Stationary peaks-over-threshold fit

# 20-year return value and standard error
result['returnValue']
# 120.3 mm
result['se_returnValue']  # return value standard error (asymptotic)
# 7.9 mm
result['logReturnProb']  # log of probability of exceeding 'returnValue=100'
# -1.98

# confidence interval on return probability for 100 mm event
np.exp(result['logReturnProb'] + np.array((-2, 2))*result['se_logReturnProb'])
# (0.0872262, 0.2200104)

Nonstationary fit with location linear in year

# change in return value (2015 - 1950) and standard error of the change
resultNS['returnValueDiff']
# -2.68 mm
resultNS['se_returnValueDiff']
# 5.37 mm

# risk ratio 2015 / 1950 for 100 mm event with confidence interval
np.exp(resultNS['logReturnProbDiff'])
# 0.88
np.exp(resultNS['logReturnProbDiff'] + np.array((-2, 2))*resultNS['se_logReturnProbDiff'])
# (0.51, 1.49)
References / Links

- Statistical methods:

- climextremes software (version 0.2.0):
  - Available via conda for python
  - Available via CRAN for R
  - Repository: [https://bitbucket.org/lbl-cascade/climextremes-dev](https://bitbucket.org/lbl-cascade/climextremes-dev)