Nonparametric estimation of the generalized propensity score based on the highly adaptive lasso

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with M. van der Laan, I. Díaz, & D. Benkeser
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Motivating example

The *observed data* unit is $O := (W, A, Y) \sim P_0 \in \mathcal{M}$:

- $W \in \mathbb{R}^d$ is a vector of baseline covariates;
- $A \in \mathbb{R}$ is a continuous-valued exposure; and
- $Y \in \mathbb{R}$ is an outcome of interest.

Let $\mathcal{M}$ be a large *semiparametric model* and for each $P \in \mathcal{M}$, define the *population intervention effect* (PIE) as

$$\Psi_\delta(P) := \mathbb{E}_P\{Y(A_\delta) - Y\} ,$$

where A_δ arises from a *stochastic* intervention.

NPSEM with static interventions

- Use a nonparametric structural equation model (NPSEM) to describe the generation of O (Pearl 2009), specifically

$$W = f_W(U_W); A = f_A(W, U_A); Y = f_Y(A, W, U_Y)$$

- Implies a model for the distribution of counterfactual random variables generated by interventions on the process.
- A *static intervention* replaces f_A with a specific value a in its conditional support $A \mid W$.
- This requires specifying a particular value of the exposure under which to evaluate the outcome *a priori*.

NPSEM with stochastic interventions

- *Stochastic interventions* modify the value A would naturally assume by drawing from a modified exposure distribution.
- Consider the post-intervention value $A^\star \sim G^\star(\cdot \mid W)$; static interventions are a special case (degenerate distribution).
- Such an intervention generates a counterfactual RV $Y_{G^\star} := f_Y(A^\star, W, U_Y)$, with distribution P_0^δ.
- We aim to estimate $\psi_{0,\delta} := \mathbb{E}_{P_0^\delta}\{Y_{G^\star}\}$, the counterfactual mean under the post-intervention exposure distribution $G^\star$.

Stochastic interventions for the causal effects of shifts

- Díaz and van der Laan (2012; 2018)'s *stochastic* interventions

$$\delta(a, w) = \begin{cases} a + \delta, & a + \delta < u(w) \quad \text{(if plausible)} \\ a, & a + \delta \geq u(w) \quad \text{(otherwise)} \end{cases}$$

- Evaluate outcome under modified *intervention distribution*: $P_\delta(g_0)(A = a \mid W) = g_0(\delta^{-1}(A, W) \mid W)$.

- Díaz and van der Laan (2012) show that $\psi_{0,\delta}$ is identified by a functional of the distribution of O:

$$\psi_{0,\delta} = \int_{\mathcal{W}} \int_{\mathcal{A}} \mathbb{E}_{P_0}\{Y \mid A = \delta(a, w), W = w\} \cdot \\ g_{0,A}(a \mid W = w) \cdot q_{0,W}(w) d\mu(a) d\nu(w)$$

- Provides a derivation based on the efficient influence function (EIF) with respect to the nonparametric model $\mathcal{M}$.

- Key innovation: loosening standard assumptions through a change in the observed intervention mechanism.

- Problem: globally altering an intervention mechanism does not necessarily respect individual characteristics.

- The authors build IPW, one-step, and TML estimators, comparing the three different approaches.

Estimation of the PIE

An *estimator* ψ_n of $\psi_0 := \Psi(P_0)$ is *efficient* if and only if

$$\psi_n - \psi_0 = n^{-1} \sum_{i=1}^{n} D^{\star}(P_0)(O_i) + o_P(n^{-1/2}) \ ,$$

where $D^{\star}(P)$ is the *efficient influence function* (EIF) of Ψ_δ with respect to the model $\mathcal{M}$ at P.

The EIF of Ψ is indexed by two key *nuisance parameters*

$$\begin{aligned} \overline{Q}_{P,Y}(A, W) &:= \mathbb{E}_P(Y \mid A, W) && \textit{outcome mechanism} \\ g_{P,A}(A, W) &:= p(A \mid W) && \textit{generalized propensity score} \end{aligned}$$

Estimation of a counterfactual mean

We'll rely on *empirical process notation* throughout:

- $P_0 f = \mathbb{E}_{P_0}\{f(O)\} = \int f(o) dP(o)$
- $P_n f = \mathbb{E}_{P_n}\{f(O)\} = n^{-1} \sum_{i=1}^{n} f(O_i)$

We can estimate the *counterfactual mean* $\Psi_\delta(P)$, using the inverse probability weighted (IPW) estimator

$$\psi_{\delta,n} = n^{-1} \sum_{i=1}^{n} \frac{g_{n,A}(\delta^{-1}(A_i, W_i) \mid W_i)}{g_{n,A}(A_i \mid W_i)} Y_i.$$

Why IPW estimators?

- IPW estimators are the oldest class of causal effect estimators.
- IPW estimators are still very commonly used in practice today.
- Easy to implement and appropriate in many settings, but...
  1. requires a correctly specified estimate of the propensity score;
  2. can be inefficient, never attaining the efficiency bound; and
  3. suffers from an (asymptotic) curse of dimensionality.

IPW estimators

The IPW estimator $\Psi_\delta(P_n, g_{n,A})$ is a solution to the score equation $P_n U_{g_{n,A}}(\Psi_\delta) = 0$, where $U_{g_A}(O; \Psi_\delta) = \frac{(g_{n,A}(\delta^{-1}(A_i, W_i)|W_i)}{g_{n,A}(A_i|W_i))} Y - \Psi(P)$:

$$\Psi_\delta(P_n, g_{n,A}) = n^{-1} \sum_{i=1}^{n} \frac{g_{n,A}(\delta^{-1}(A_i, W_i) \mid W_i)}{g_{n,A}(A_i \mid W_i)} Y_i.$$

- Consistency and convergence rate of IPW relies on those same properties of the propensity score estimator $g_{n,A}$.
- Generally, finite-dimensional (i.e., parametric) models are not flexible enough to consistently estimate $g_{0,A}$.

- Use of parametric models for constructing g_n results in bias and confidence intervals with coverage zero asymptotically.
- Better served by using flexible, data-adaptive estimators, but this brings its own challenges, e.g., achieving quarter-rate convergence.

Nonparametric conditional density estimation

- Our IPW estimator require the generalized propensity score, at both $g_A(A \mid W)$ and $g_A(\delta^{-1}(A, W) \mid W)$.
- There is a rich literature on density estimation, we follow the approach first explored in Díaz and van der Laan (2011).
- To build a conditional density estimator, consider

$$g_{n,A,\alpha}(a \mid W) = \frac{\mathbb{P}(A \in [\alpha_{t-1}, \alpha_t) \mid W)}{\alpha_t - \alpha_{t-1}},$$

  for $\alpha_{t-1} \leq a < \alpha_t$.
  - This is a classification problem, where we estimate the probability that a value of A falls in a bin $[\alpha_{t-1}, \alpha_t)$.
  - The choice of the tuning parameter t corresponds roughly to the choice of bandwidth in classical kernel density estimation.

Nonparametric conditional density estimation

- Díaz and van der Laan (2011) propose a reformulation of this classification approach as a set of hazard regressions.
- To effectively employ this proposed reformulation, consider

$$\mathbb{P}(A \in [\alpha_{t-1}, \alpha_t) \mid W) = \mathbb{P}(A \in [\alpha_{t-1}, \alpha_t) \mid A \geq \alpha_{t-1}, W) \times \Pi_{j=1}^{t-1}\{1 - \mathbb{P}(A \in [\alpha_{j-1}, \alpha_j) \mid A \geq \alpha_{j-1}, W)\}$$

  - Likelihood may be re-expressed as the likelihood of a binary variable in a repeated measures data structure.
  - Specifically, the observation of O_i is repeated as many times as intervals $[\alpha_{t-1}, \alpha_t)$ are prior to the interval to which A_i falls, and the indicator variables $A_i \in [\alpha_{t-1}, \alpha_t)$ are recorded.

Curse of dimensionality

Goal: Construct nuisance parameter *estimators* that are *consistent* and *converge faster* than $n^{-1/4}$ under *minimal assumptions*.

Challenging for moderately large d, i.e., *curse of dimensionality*.

For example, consider *kernel regression* with bandwidth h and kernels orthogonal to polynomials in W of degree k.

- Assume parameter is k times *differentiable*.
- Optimal bandwidth $O(n^{-1/(2k+d)})$
- Optimal convergence rate $O(n^{-k/(2k+d)})$

Curse of dimensionality

Broadly, *two approaches* for handling the *curse of dimensionality*.

[1] Enforce (strong) *smoothness assumptions* on model space.

- No guarantee of *consistency*

[2] Ensemble machine learning, e.g., *Super Learning*

- No guarantee of *quarter rates*

An important class of functions

Consider space of *cadlag* functions with *finite variation norm*.

Def. cadlag = *left-hand continuous* with *right-hand limits*

Variation norm Let $\theta_s(u) = \theta(u_s, 0_{s^c})$ be the *section* of θ that sets the coordinates in *s equal to zero*.

The *variation norm* of θ can be written:

$$|\theta|_v = \sum_{s \subset \{1,\ldots,d\}} \int | \, d\theta_s(u_s) \, |,$$

where $x_s = (x(j) : j \in s)$ and the sum is over all subsets.

Variation norm

We can represent the function θ as

$$\theta(x) \quad = \quad \theta(0) + \sum_{s \subset \{1,\ldots,d\}} \int I(x_s \geq u_s) d\theta_s(u_s),$$

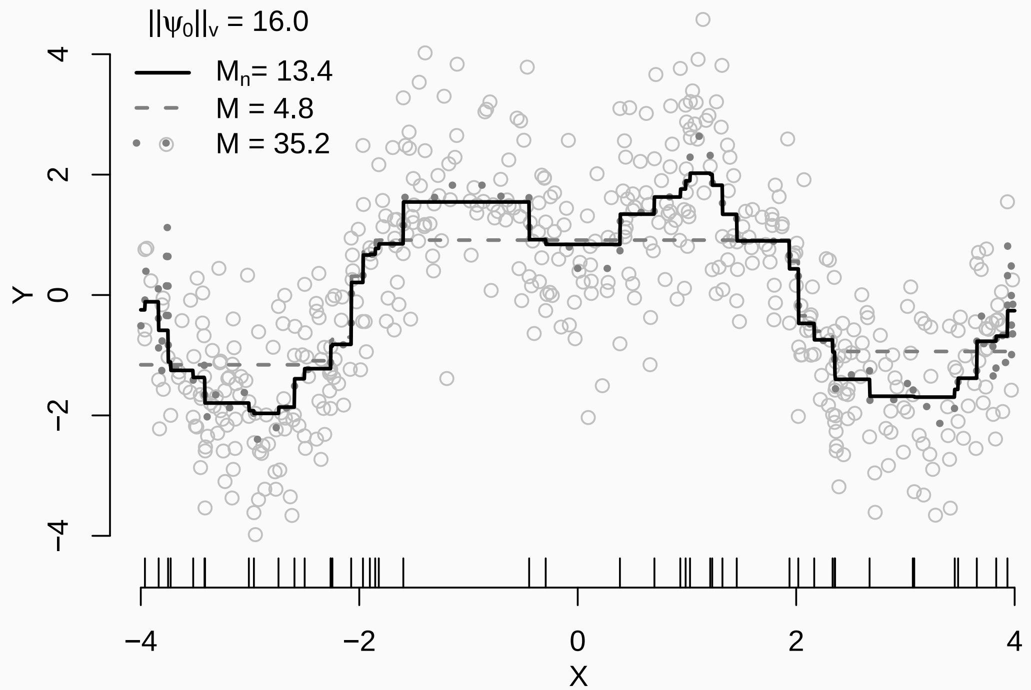
For discrete measures $d\theta_s$ with *support points* $\{u_{s,j} : j\}$ we get a *linear combination* of indicator *basis functions*:

$$\theta(x) = \theta(0) + \sum_{s \subset \{1,\ldots,d\}} \sum_j \beta_{s,j} \theta_{u_{s,j}}(x),$$

where $\beta_{s,j} = d\theta_s(u_{s,j})$, $\theta_{u_{s,j}}(x) = I(x_s \geq u_{s,j})$, and

$$|\theta|_v = \theta(0) + \sum_{s \subset \{1,\ldots,d\}} \sum_j |\beta_{s,j}|.$$

HAL illustration



Convergence rate of HAL

We have

$$|\theta_{n,M} - \theta_{0,M}|_{P_0} = o_P(n^{-(1/4+\alpha(d)/8)}),$$

where $\alpha(d) = 1/(d+1)$.

Thus, if we select $M > |\theta_0|_v$, then

$$|\theta_{n,M} - \theta_0|_{P_0} = o_P(n^{-(1/4+\alpha(d)/8)}) \ .$$

Due to oracle inequality for the cross-validation selector M_n,

$$|\theta_{n,M_n} - \theta_0|_{P_0} = o_P(n^{-(1/4+\alpha(d)/8)}) \ .$$

Improved rate (Bibaut and van der Laan 2019):

$$|\theta_{n,M_n} - \theta_0|_{P_0} = o_P(n^{-1/3} \log(n)^{d/2}) \ .$$

- The goal was to construct nuisance parameter estimators that are *consistent* and *converge faster* than $n^{-1/4}$ under *minimal assumptions*.
- We have assumed enough *smoothness* that HAL will *converge faster* than $n^{-1/4}$, but retain enough flexibility that *consistency* is also preserved.

HAL estimate of $g_{0,A}$

If the nuisance functional $g_{0,A}$ is cadlag with finite sectional variation norm, logit g can be expressed (Gill et al. 1995):

$$\operatorname{logit} g_\beta = \beta_0 + \sum_{s \subset \{1,\ldots,d\}} \sum_{i=1}^{n} \beta_{s,i}\phi_{s,i},$$

where $\phi_{s,i}$ is an indicator basis function.

The loss-based HAL estimator β_n may then be defined as

$$\beta_{n,\lambda} = \underset{\beta: |\beta_0| + \sum_{s \subset \{1,\ldots,d\}} \sum_{i=1}^{n} |\beta_{s,i}| < \lambda}{\arg\min} P_n L(\operatorname{logit} g_\beta),$$

where $L(\cdot)$ is an appropriate loss function.

Denote by $g_{n,\lambda} \equiv g_{\beta_{n,\lambda}}$ the HAL estimate of $g_{0,A}$.

Undersmoothing HAL for IPW estimation

Beyond the cross-validation selector's choice of λ_n, we propose

1. EIF-based: choose λ_n s.t.

$$\lambda_n = \arg\min_{\lambda} |P_n D^\star(g_{n,A,\lambda}, \overline{Q}_{n,Y})|,$$

where $\overline{Q}_{n,Y}$ is an estimate of $\overline{Q}_{0,Y}(A, W)$.

2. Plateau-based: choose λ_n as the first in $\lambda_1, \ldots, \lambda_K$ s.t.

$$|\psi_{n,\lambda_{j+1}} - \psi_{n,\lambda_j}|_{j=1}^{K-1} \leq Z_{(1-\alpha/2)}[\sigma_{n,\lambda_{j+1}} - \sigma_{n,\lambda_j}]_{j=1}^{K-1},$$

where σ_{n,λ_j} is a variance estimate at λ_j.

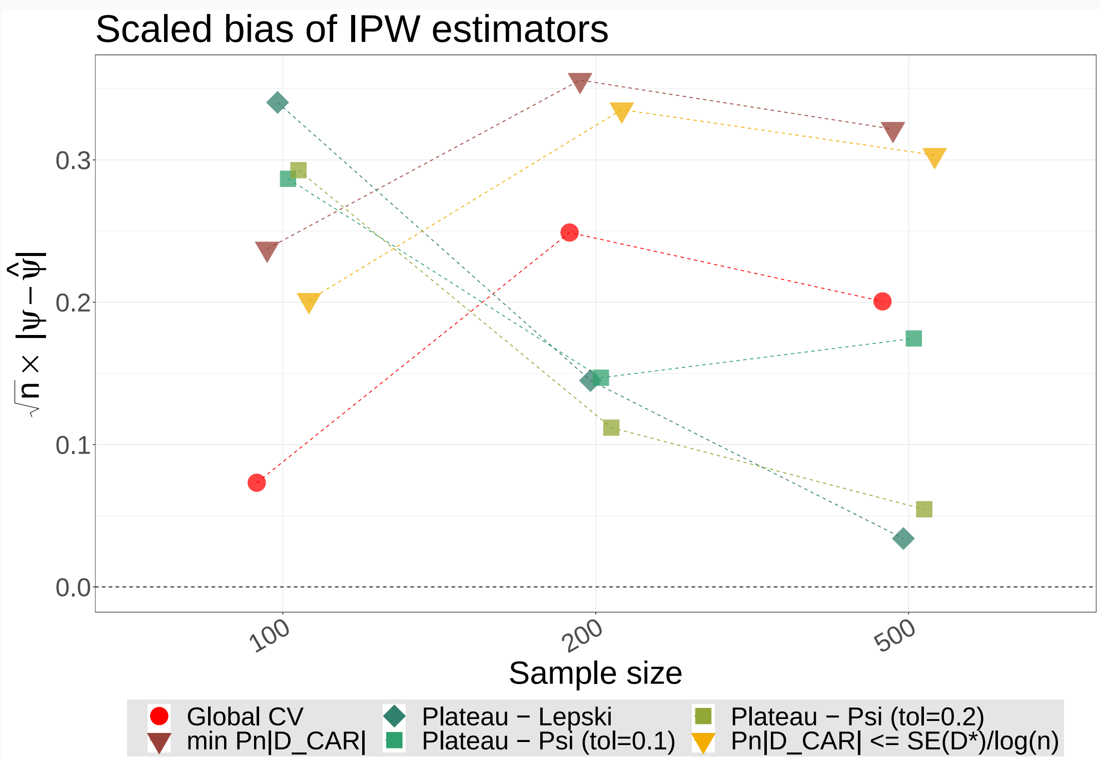
3. Plateau-based: choose λ_n as the first in $\lambda_1, \ldots, \lambda_K$ s.t.

$$\left[\frac{|\psi_{n,\lambda_{j+1}} - \psi_{n,\lambda_j}|}{\max_j |\psi_{n,\lambda_{j+1}} - \psi_{n,\lambda_j}|}\right]_{j=1}^{K-1} \leq \kappa$$

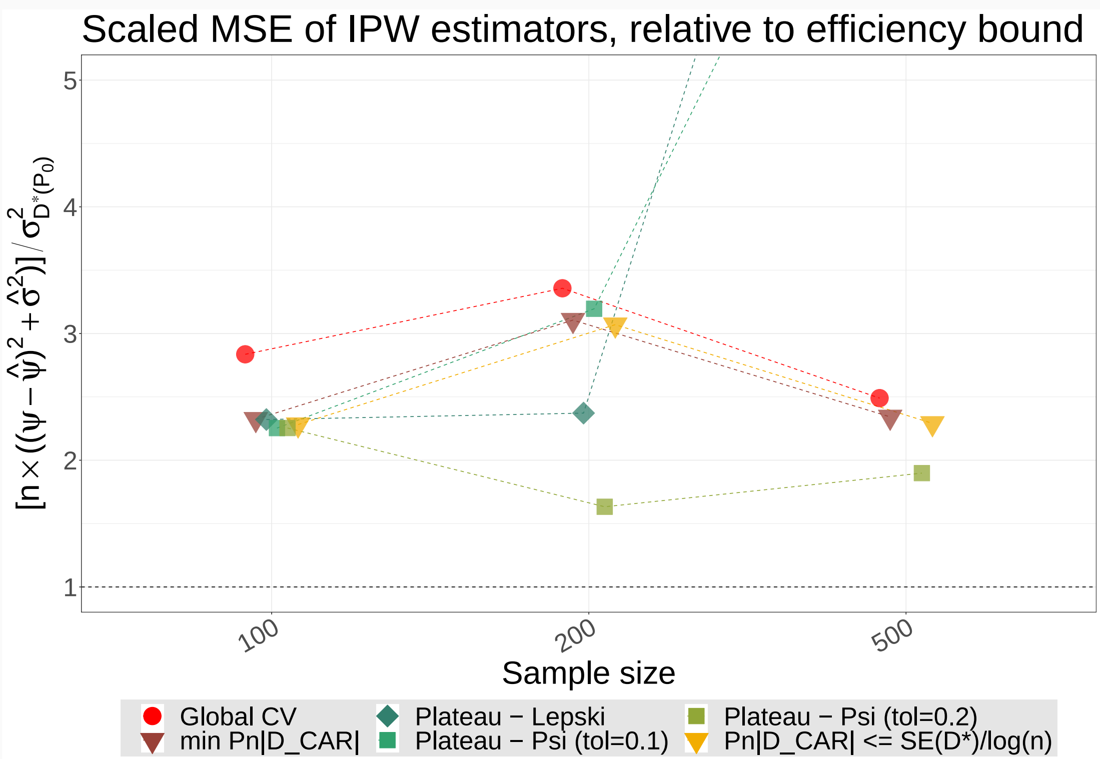
for an arbitrary tolerance level κ.

- We show that inverse probability weighted estimators can be asymptotically (nonparametric) efficient when the propensity score is estimated using an undersmoothed HAL estimator.
- Cross-validation is often used to tune the HAL fit (i.e., the sectional variation norm).
- By contrast, we undersmooth the HAL fit by selecting a tuning parameter that is larger than the one selected by cross-validation.
- Undersmoothing is done in a targeted manner to guarantee asymptotically linearity of the estimator.

Simulation: Bias



Simulation: MSE



The big picture

1. Unlike classical IPW estimators, ours avoid the asymptotic curse of dimensionality and are asymptotically efficient;
2. Our approach leverages flexible conditional density estimation for initial generalized propensity score estimates; and
3. In contrast with doubly robust estimators, our estimators can be formulated without the form of the EIF.

Thank you!

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Appendix

From the causal to the statistical target parameter

Assumption 1: *Stable Unit Treatment Value (SUTVA)*

- $Y_i^{\delta(a_i, w_i)}$ *does not depend on* $\delta(a_j, w_j)$ *for* $i = 1, \ldots, n$ *and* $j \neq i$*, or lack of interference (Rubin 1978; 1980)*
- $Y_i^{\delta(a_i, w_i)} = Y_i$ *in the event* $A_i = \delta(a_i, w_i)$, $i = 1, \ldots, n$

Assumption 2: *Ignorability*

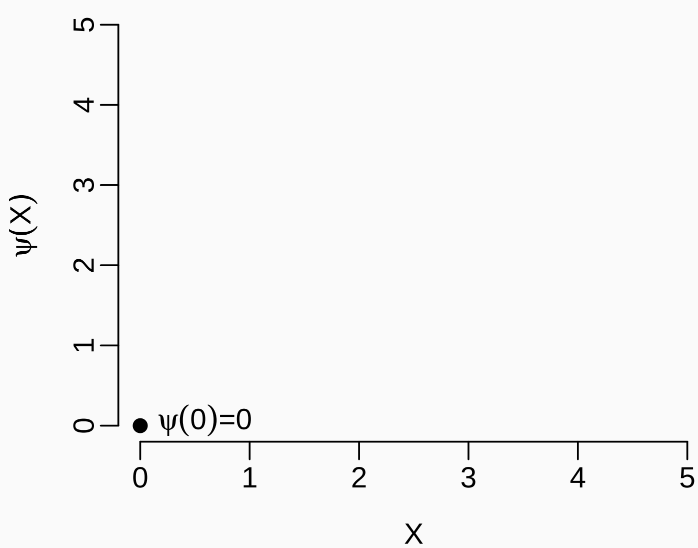
$A_i \perp\!\!\!\perp Y_i^{\delta(a_i, w_i)} \mid W_i$, *for* $i = 1, \ldots, n$

Assumption 3: *Positivity*

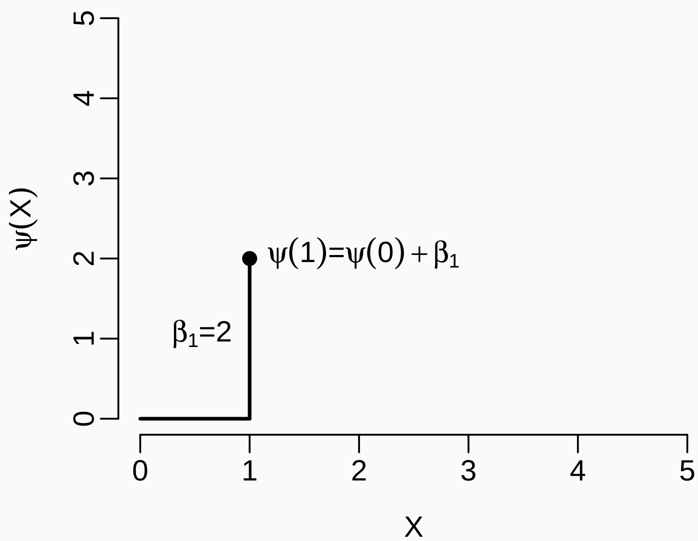
$a_i \in \mathcal{A} \implies \delta(a_i, w_i) \in \mathcal{A}$ *for all* $w \in \mathcal{W}$, *where* $\mathcal{A}$ *denotes the support of* A *conditional on* $W = w_i$ *for all* $i = 1, \ldots n$

- This positivity assumption is not quite the same as that required for categorical interventions.
- In particular, we do not require that the intervention density place mass across all strata defined by W.
- Rather, we merely require the post-intervention quantity be seen in the observed data for given $a_i \in \mathcal{A}$ and $w_i \in \mathcal{W}$.

HAL illustration



HAL illustration



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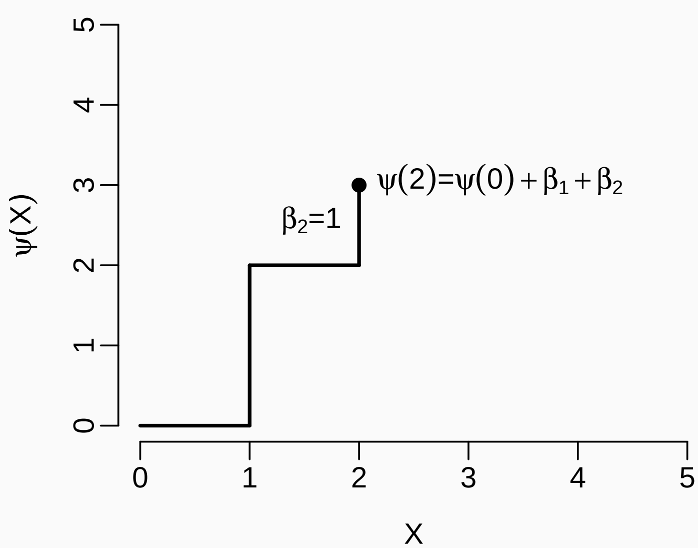

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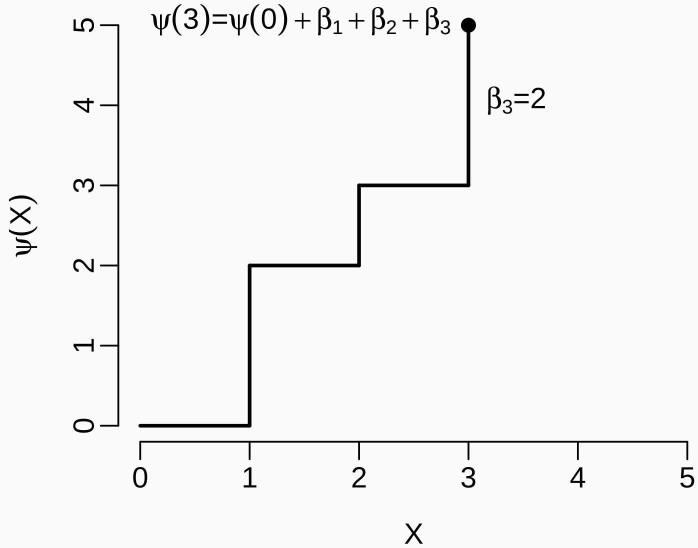

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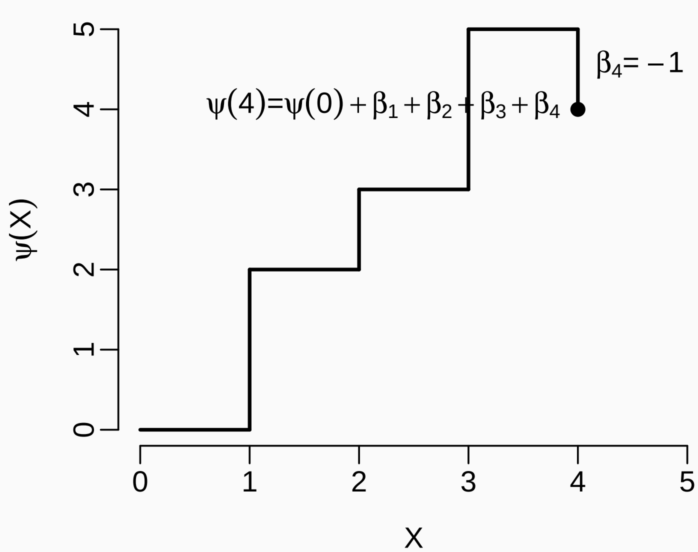

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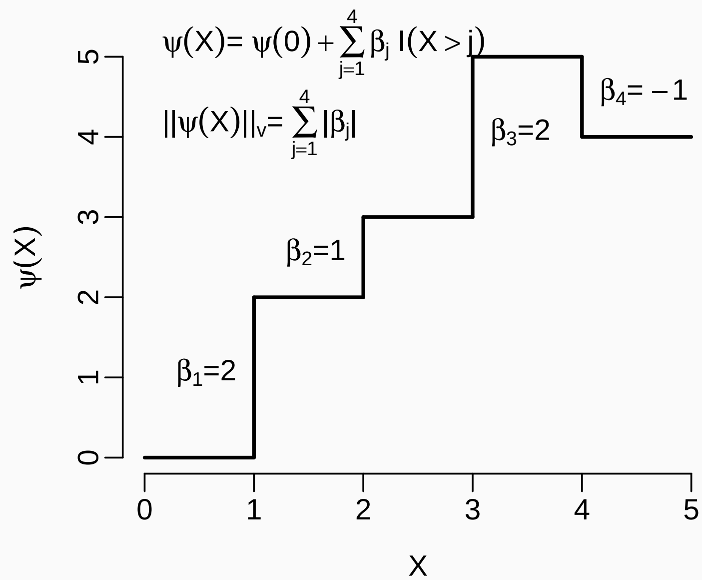

Literature: Haneuse and Rotnitzky (2013)

- *Proposal:* Characterization of stochastic interventions as *modified treatment policies* (MTPs).
- Assumption of *piecewise smooth invertibility* allows for the intervention distribution of any MTP to be recovered:

$$g_{0,\delta}(a \mid w) = \sum_{j=1}^{J(w)} I_{\delta,j}\{h_j(a,w), l\} g_0\{h_j(a,w) \mid l\} h_j'(a,w)$$

- Such intervention policies account for the natural value of the intervention A directly yet are interpretable as the imposition of an altered intervention mechanism.
- Identification conditions for assessing the parameter of interest under such interventions appear technically complex (at first).

- Shifts of the form $\delta(A, W)$ are considerably more interesting since these are realistic intervention policies.
- Example: consider an individual with an extremely high immune response but whose baseline covariates W suggest we shift the response still higher. Such a shift may not be biologically plausible (impossible, even) but we cannot account for this if the shift is only a function of W.
- The authors build IPW, outcome regression, and non-iterative doubly robust estimators, as well as an approach based on MSMs.
- Piecewise smooth invertibility: This assumption ensures that we can use the change of variable formula when computing integrals over A and it is useful to study the estimators that we propose in this paper.

Literature: Young et al. (2014)

- Establishes equivalence between g-formula when proposed intervention depends on natural value and when it does not.
- This equivalence leads to a sufficient positivity condition for estimating the counterfactual mean under MTPs via the same statistical functional studied in Díaz and van der Laan (2012).
- Extends earlier identification results, providing a way to use the same statistical functional to assess $\mathbb{E}Y_{\delta(A,W)}$ or $\mathbb{E}Y_{\delta(W)}$.
- The authors also consider limits on implementing shifts $\delta(A, W)$, and address working in a longitudinal setting.

Literature: Díaz and van der Laan (2018)

- Builds on the original proposal, accomodating MTP-type shifts $\delta(A, W)$ proposed after their earlier work.
- To protect against positivity violations, considers a specific shifting mechanism:

$$\delta(a, w) = \begin{cases} a + \delta, & a + \delta < u(w) \\ a, & \text{otherwise} \end{cases}$$

- Proposes an improved TMLE algorithm, with a single auxiliary covariate for constructing the TML estimator.

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