Social Choice and Social Networks

Some non martingale learning models

Draft -

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The Gale Kariv Model

- Bayesian setup – state of the word S = +/- with prob. ½.
- independent private signals s(1),…,s(n).
- s follows F_+ / F_- where
- $x(s) := dF(+)(s)/dF(-)(s) \in (0,\infty)$ for all s.
- Assume x is non atomic and has a density.
- Declaration at time t:
- $d(v,t) := \text{sgn } E[f \mid s(v), d(w,s) : w \in N(v), s < t]$

The Gale Kariv Model

- Declaration at time t:
- $d(v,t) := \text{sgn}\, E[\, S \mid s(v), d(w,s) : w \in N(v),\ s < t]$

- Let $F(v,t) := \text{Gen}\{s(v), d(w,s) : w \in N(v),\ s < t\}$, $F(v,t) \rightarrow F(v)$
- Let $f(v,t) = E[S \mid F(v,t)] \rightarrow f(v) = E[S \mid F(v)]$

- Note if $f(v)>0$ then declarations of v converge to +.

- Claim(Gale, Kariv): Could not be that $f(v)>0$ and $f(w)<0$ in a strongly connected graph.

- We will try to prove it …

The Gale Kariv Model

- Let $F(v,t) := Gen\{s(v), d(w,s) : w \in N(v), s < t\}$, $F(v,t) \to F(v)$
- Let $G(v,t) := Gen\{d(v,s) : s < t\}$, $G(v,t) \to G(v)$
- Let $f(v,t) = E[S \mid F(v,t)] \to f(v) = E[S \mid F(v)]$
- Let $g(v,t) = E[S \mid G(v,t)] \to g(v) = E[S \mid G(v)]$

- Note that $f(v)>0 \Rightarrow d(v,t) \to d(v) = +$ and
  $f(v)<0 \Rightarrow d(v,t) \to d(v) = -$
  define $d(v)$ arbitrarily otherwise.
- Let $e(v) = +$ if $d(v,t) \to +$ and otherwise $e(v)=-$.

The Gale Kariv Model

- Note that $f(v)>0 \Rightarrow d(v,t) \rightarrow d(v) = +$ and
  $f(v)<0 \Rightarrow d(v,t) \rightarrow d(v) = -$
  define d(v) arbitrarily otherwise.
- Let $e(v) = +$ if $d(v,t) \rightarrow +$ and otherwise $e(v)=-$.
- <u>Claim:</u> The functions d(v),e(v) both maximize the predication probability of the signal given F(v).
- Let $G(v,w,t) := \mathrm{Gen}\{d(v,s), d(w,s) : s < t\}$, v,w nbrs
- Let c(v,w) maximize the prediction probability given G(v,w).
- <u>Claim:</u> Unless $f(v)=0$ or $f(w)=0$ it holds that $d(v)=e(v)=c(v,w)$.
- <u>Cor:</u> Unless $f(v)=0$ or $f(w)=0$ we have $d(v)=d(w)$.

- <u>Remains to prove</u>: f(v) and f(w) cannot be 0.

The Gale Kariv Model

- Claim: $f(v)$ and $f(w)$ cannot be 0.
- Pf: $P[+ \mid F(v)] / P[- \mid F(v)] = x(s)\, P[+ \mid G(v)] / P[- \mid G(v)]$
- So: $g(v) = (x\, g(v)+g(v)-1)/(x\, g(v)-g(v)+1)$
-
- Note further g can take only countably many values since:
- Either g=0.
- $g>0 \Rightarrow \exists$ an n such that $d(v,t)=+$ for all $t> n$
- $g<0 \Rightarrow \exists$ an n such that $d(v,t)=+$ for all $t> n$

- For each of the values that g takes for f=0 x has to take some special value which has probability 0.

The Gale Kariv Model - Challenges

- Not known: How long does it take to converge?
- Not known: Do big networks aggregate well?
- Very hard model to tackle analytically.

Truncated information on the complete graph

- Model in M-Tamuz-10 :=
- Gale Kariv Model on the complete graph.

- <u>Results:</u>

- All agents converge to the same outcome.

- Their calculations are efficient.

- Process is statistically more accurate than any process involving finite # of rounds of voting.

- <u>Technical assumption:</u>
- $x(s) := dF(+)(s)/dF(-)(s) \in (0,\infty)$ for all s.
- Assume x is non atomic and has a density.

Truncated information on the complete graph

- Example of repeated voting: electing a pope.
- Stops in 2/3 majority.

From the Apostolic Constitution of Pope Paul VI (1975, II, ii, 42):

"Conclave" means the carefully determined place, a kind of sacred retreat, where, after asking the Holy Spirit to enlighten them, the cardinal electors choose the Supreme Pontiff, and where the cardinals and other officials and staff, together with the conclavists, if there be any, remain day and night until the election is complete, and do not communicate with persons and things outside, in accordance with the following modalities and norms …

Voting process as viewed from outside

- Suppose we view the process from the outside.
- What can we learn about $x(v)$, $v \in V$?
-

•At round 1: we know that $x(v) > 0$ for those voting + and $x(v) < 0$ for those voting 0.

- This information is obviously also known to all voters.
- Before round 2, player v will calculate:
- $\ln P[+]\backslash P[-] = \ln x(v) + \sum \{ y(w) : w \neq v\}$
- where $y(w) = \ln P[x(w) > 0 \mid +] - \ln P[x(w) > 0 \mid -]$ assuming we know $x(w) > 0$ and
- $y(w) = \ln P[x(w) < 0 \mid +] – \ln P[x(w) < 0 \mid -]$ assuming we know $x(w) < 0$ and
- The vote of $v = \mathrm{sgn}(\ln x(v) + \sum \{ y(w) : w \neq v\})$
- Gives new bounds on $x(v)$.

Voting process as viewed from outside

- At round t: we know that $x(v) \in (a(v,t),b(v,t))$
- This information is obviously also known to all players.
- Before round 2, player v will calculate:
- $\ln P[+]\backslash P[-] = \ln x(v) + \sum \{y(w,t) : w \neq v\}$
- where $y(w,t) = \ln P[x(w) \in (a(w,t),b(w,t)) \mid + / -]$
- This gives new bounds on x(v).

- This describes the algorithm of the agents.
- Work out an example.

Voting process as viewed from outside

- Thm: With probability 1 all agents converge to the same vote.

- Pf idea:
- Let y(w) = lim y(w,t).
- Assume first $y = \sum_w y(w) > 0$ (outsider: + eventually)
- No convergence $\Rightarrow \exists$ v and infinite # of t s.t.:
- $\sum$ y(w,t) > y/2, v votes -.
- At each such vote we update b(v,t) := b(v,t) – y/2.
- But $x \in$ [a(v,t),b(v,t)] for all t.
- The case y < 0 is treated similarly.

Voting process as viewed from outside

- Thm: With probability 1 all agents converge to the same vote.

- Pf idea:
- Let y(w) = lim y(w,t).
- Assume now y := $\sum$ y(w) = 0 (outside viewer not sure)
- May assume $\sum$ x(v) $\neq$ 0.
- Means there must be v with (a(v,t),b(v,t)) not converging to x(v).
- Suppose v votes – infinitely often
- This means a(v,t) is updated infinitely many times:
- a(v,t+1) = max{a(v,t),x(v,t)- $\sum$ y(w,t)}
- But this means a(v,t) $\rightarrow$ x(v).

Other facts

- Thm: In the case of biased signals – convergence to the correct action w.h.p after 2 rounds.

- Pf idea: After vote 1 individuals see n independent signals on the state of the world.

- Thm: The more rounds the better.

- Pf idea: The probability of voter i being correct increases with time.

Heuristic Dynamics on Networks

- So far discussed various Bayesian models for opinion updates.

- Now:
- Discuss simpler heuristic models of repeated voting.

- Why other models?

Heuristic Dynamics on Networks

- This time:
- Discuss simpler heuristic models of repeated voting.

- Why other models?

- Real "agents" are probably not fully Bayesian. Instead they apply some heuristics (but which?).

- Simpler update rule may lead to more complete analysis in understanding various features of the behavior.

The Two Models We'll discuss

• DeGroot model: repeated averaging of probabilities of self and neighbors.

• Repeated Majority: Taking repeated majority of opinions of self and neighbors.

The DeGroot Model

• DeGroot model: repeated averaging of probabilities of self and neighbors.

• Formal definition: n individuals denoted 1,…,n.
• At time 0 some initial beliefs: p(i,0) for i $\in$ [n].
• Averaging weights: w(i,j) $\geq$ 0 satisfy $\sum_j$ w(i,j) =1
• Update rule: p(i,t+1) = $\sum_j$ w(i,j) p(j,t)
• In matrix notation: p(t+1) = W p(t)

• Linear model, linear updates …
• Introduced by De Groot (Penn Statistician, 1931-1989) in 1974.

The DeGroot Model - Examples

- In matrix notation: $p(t+1) = W\,p(t)$
- Ex 1: $W = I$ ($w(i,i) = 1$ for all i) ?
- Ex 2: $w(i,j) = 1/n$ for all i,j ?
- Ex 3: $w(i,j) = 1/3$ for $i-j \bmod n \in \{0,1,n-1\}$?
- Ex 4: $w(i,j) = ½$ for $i-j \bmod n \in \{1,n-1\}$?
- <u>Ex 5:</u> $w(i,n) = w(i,i) = ½$ for $i < n$ and $w(n,j) = 1/n$ for all j.

The DeGroot Model - Convergence

- In matrix notation: p(t+1) = W p(t)

- p(t+1,i) = E(i,t) p(X) where

- E(i,t) denote expectation according to P(i,t)

- P(i,t) := Markov Chain according to W started at i and run for t steps.

Some Markov Chain Background

- The total variation distance between two distributions P,Q is given by $0.5 \sum_x |P(x)-Q(x)|$

- Def: A finite Markov chain given by transition P is called ergodic if there exists a k such all entries of P^k are positive.

The DeGroot Model - Convergence

• In matrix notation: $p(t+1) = W\, p(t)$
• $p(t+1,i) = E(i,t)\, p(X)$ where $E(i,t)$ denote expectation according to $P(i,t)$:= M.C. according to W started at i and run for t steps.

• If W corresponds to an ergodic chain we know that:
• there exist a unique stationary distribution π.
• and $|P(i,t) - \pi|_{TV} \rightarrow 0$.
• So $| E(i,t)\, p - E(\pi)\, p| \rightarrow 0$
• In other words: all probabilities converge to the same value which is $E(\pi)\, p = \sum_v \pi(v)\, p(v)$.

What is π ?

- Claim: If W is an ergodic random walk on an undirected graph then:
- $\pi(i) = \deg(i) \,/\, \sum_j \deg(j)$.

- Conclusion: Importance determined by # of neighbors.

The DeGroot Model - Examples

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Rate of Convergence

- In the theory of M.C. there are many ways to measure the rate of convergence of a chain.
- We can directly use any of them to obtain bounds on the rate of convergence to the common opinion via:
- $\max_i | E(i,t)\, p - E(\pi)\, p| \leq \max_i | P(i,t) - \pi|$

- These include:

- Spectral gap bounds:
- Conductance bounds
- etc. (Log-Sobolev bounds, coupling bounds …)

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- These include:

- Spectral gap bounds:
- Let g = smallest non-zero eigen-value of I-1/2(P + P*)
- Then: by time $t = s\,(1+\max_i \ln(1/\pi(i)))/g$ we have:
- $\max_i | P(i,t) - \pi| \leq \exp(-s)$.

Rate of Convergence

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- These include:

- <u>Condoctance bounds:</u>
- $\pi(\partial A) := \sum \{w(x,y)\, \pi(x) + w(y,x)\, \pi(y) : (x,y) \in (A,A^c)\}$
- $I = \min \{Q(\partial A)/ \pi(A) : \pi(A) \leq ½\}$.
- Then $g \geq I^2/8$ (Cheeger's inequality) and therefore
- By time $t = 8\, s\, (1+\max_i \ln(1/\pi(i)))/I^2$ we have:
- $\max_i | P(i,t) - \pi| \leq \exp(-s)$.

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- By time $t = 8\ s\ (1+\max_i \ln(1/\pi(i)))/I^2$ we have:
- $\max_i |\ P(i,t) - \pi| \leq \exp(-s)$.

- This means if there are no "isolated communities" then convergence is quick.

- In the minimum suffices to look at connected sets.

The DeGroot Model – Convergence rate Examples

- In matrix notation: $p(t+1) = W\, p(t)$
- Ex 1: $W = I$ ($w(i,i) = 1$ for all i) ?
- Ex 2: $w(i,j) = 1/n$ for all i,j ?
- Ex 3: $w(i,j) = 1/3$ for $i-j \bmod n \in \{0,1,n-1\}$?
- Ex 4: $w(i,j) = ½$ for $i-j \bmod n \in \{1,n-1\}$?
- Ex 5: $w(i,n) = w(i,i) = ½$ for $i < n$ and $w(n,j) = 1/n$ for all j.

Some non ergodic examples

- Ex 1: Cycle $W(i,i+1) = 1$.
- Ex 2: A complete bi-partite graph.
- Ex 3: An erogdic chain with followers.
- Todo: add details and figures.

The effect of cheating

- A basic question:
- What happens if somebody cheats? Can they convince the rest of the group to reach whatever value they want?

- Claim: If the chain is ergodic and there is one cheater by repeatedly stating a fixed value all opinions will converge to that value.

- Pf: Choose t so that f(t,1) = x and f(t,i) = \sum w(i,j) f(0,j) + w(i) x and all w's are positive.

- The map f(0,.) → f(t,.) is a contraction with x being the fixed point.
- Q: What about two cheaters? Hint: Harmonic functions ..

The iterated majority model

- One model that avoids this “cheating effect” is
- repeated majority.
- Here the original signals s(i,0) are + / -
- There are weights w(i,j) and:
- $s(i,t+1) = \mathrm{sign}(\sum_j w(i,j)\, s(j,t))$
-
- We will assume that the w are chosen in such a way that there are never ties.

- A special case: a graph G and:
- Nodes of odd degrees take majority of neighbors.
- Nodes of even degrees take majority of neighbors and self.

- Non-linear analogue of De-Groot model.

Iterated Majority - Examples

- Ex 1: $W = I$ ($w(i,i) = 1$ for all i) ?

- Ex 2: $w(i,j) = 1$ for all i,j and n odd.

- Ex 3: $w(i,j) = 1$ for $i-j \bmod n \in \{0,1,n-1\}$?

- Ex 4: $w(i,j) = 1$ for $i-j \bmod n \in \{1,n-1\}$?

- <u>Ex 5: </u>$w(i,n) = 1$ for $i < n$ and $w(n,j) = 1$ for all j, n odd.

Basic questions

- Does it converge?
- If so to what?
- If doesn’t converge – what can we say about the dynamics?
- Many of these questions are open!

Convergence?

- Bi-partite graphs can be used to construct examples of non-convergence.

- In directed graph can get to cycle of length n or even exponential in n.

- What is the longest possible cycle in an undirected graph?

- In arbitrary starting condition hard to ask what it converges to …

Convergence for biased signals

- What happens for biased signals?

- Some partial results are known for asynchronous dynamics:

- Poisson clock for every node – then update to majority of neighbors (with (1/2,1/2)) transitions if even.

- Results in statistical physics: For infinite dimensional grids (dimensions 2 and higher) and regular trees there exists $q < 1$ s.t. if initial signals are i.i.d. $p > q$ then with probability 1 all converge to +.
- Howard: the exists $p>1/2$ for which on infinite 3-regular tree there is no convergence to all +.

Convergence for biased signals

- Example: Finite binary trees.
- Let T_n = n level binary tree (with degree of root = 3).

Convergence for biased signals

- Example: Finite binary trees.
- Let T_n = n level binary tree (with degree of root = 3).

- Claim: Let $0.5 < p < 1$. Then the probability of convergence to all + $\rightarrow$ 0 as $n \rightarrow \infty$.

- Pf: Suppose a node at level (n-1) is – and has its two children –s then will never change to +.

- Question: What is an example of families of graphs for which majority dynamics aggregate well.

Convergence for biased signals

- <u>Question:</u> What is an example of families of graphs for which majority dynamics aggregate well.
- Def: A graph $G=(V,E)$ is an (e,s) expander if
- $\forall S \subset |V|, |S| \leq s |V|$:
- $|\{ v \in S^c : \exists u \in S, (u,v) \in E\}| \geq e |S|$.

Convergence for biased signals

- <u>Question:</u> What is an example of families of graphs for which majority dynamics aggregate well.
- Def: A graph $G=(V,E)$ is an (e,s) expander if
- $\forall\, S \subset |V|,\ |S| \leq s\,|V|$:
- $|\{ v \in S^c : \exists\, u \in S,\ (u,v) \in E\}| \geq\ e\,|S|$.

- <u>Claim (won't prove):</u> There exists $a>0$ such that for large k and n a random k-regular graph G on n vertices it holds that G is a $(0.75\,k,\ a/k)$ expander with high probability.

- <u>Claim:</u> For graphs as above if $p>1-a/2k$ the graph aggregates well with high probability.
- In fact we'll prove that if at time 0 the fraction of + is at least $1-a/k$ then the dynamics will converge to all +.
- Proof is from Kanoria-Montanari-10

Convergence for biased signals

Claim: For graphs as above if $p>1-a/2k$ the graph aggregates well with high probability.

- In fact we'll prove that if at time 0 the fraction of + is at least $1-a/k$ then the dynamics will converge to all +.
- Proof is from Kanoria-Montanari-10

- Pf:?

Convergence for biased signals

<u>Claim</u> If at time 0 the fraction of + is at least 1-a/2k then the dynamics will converge to all +.

- <u>Pf:</u>
- We will show that # of – contracts by ¾ at each iteration.
- Let S(-) := set of –, s :=|S(-)| $\leq$ a n / k.
- n(+) := # of +'s with at most k/2 nbrs in S(-)
- n(-) := # of +'s with at least k/2 nbrs in S(-).
- l := # of edges from S(-) to itself.
- Then by expansion: n(+) + n(-) $\geq$ 0.75 k s
- n(+) + k/2 n(-) + 2l $\leq$ k s.
- So: (0.5 k – 1) n(-) + 2l $\leq$ 0.25 ks.
- So for k large enough: 0.5 k n(-) + 2l $\leq$ 0.3 k s
- So: n(-) + 2l / (k/2) $\leq$ 0.6 s
- So # of –'s in next iteration at most 0.6s.

Convergence to the same action - conclusions

- Many examples where it is easy to establish it is not the case that all voters converge to the same vote.

- In some rare examples possible to show convergence to the same vote.

- Arguments use strong bias and “expansion”.

Asking a different question

- So far we analyzed if repeated majority results in achieving consensus.

- Since we know in real life consensus is often not reached …

- Perhaps we should aim for less:

- What is the effect of repeated majority on the outcome of the vote?

Asking a different question

- Setup:

- Originally voters receive (+,-) signal which is correct with probability $p > ½$.

- Optimal fair voting: majority of original signals.

- Instead:

- voters converse/learn and change their opinions for 100 days

- Each day update to the view of the majority of the people they appreciate.

Example 1: “US Media”

- There are 11 media outlets.

- n=200,000,000 voters.

- Each media outlet takes a majority vote among 1-11 other media outlets.

- Each voter takes a majority vote among 1-3 friends and 5-11 media outlets.

- Repeat for d days.

- Then a majority vote among all voters.

- Does aggregation hold?

Example 1: "US Media"

- There are 101 media outlets.
- n=200,000,000 voters.
- Each media outlet takes a majority vote among 1-101 other media outlets.
- Each voter takes a majority vote among 1-10 friends and 30-50 media outlets.

Q: Does aggregation hold if repeat for d days?

- A: No! Enough that media gets the wrong signal on first round.
- Q: Does this happen in "real life"? topics not covered by media?
- <u>Note: </u>Possible to construct more realistic examples.
- In particular suffices that positive fraction of population give more weight to media,

Comments on Example 1: “US Media”

•<u>Note: </u>Possible to construct more realistic examples.
• In particular suffices that positive fraction of population give more weight to media, for example:

• 10% of people give more weight do media then to their friends.
• 10% of people give more weight to the media and people in previous group than to the rest of their friends.
• etc. (for say 60% of voters).

• Can we make an example like this when the media also takes the opinions of people into account? (an undirected example)?

Example 1: “US Media (2)”

- Can we make an example like this when the media also takes the opinions of people into account?

- Yes! For example …

- Suppose there is 1 media outlet.
- n voters partitioned into pairs.
- outlet takes majority over all voters.
- voters take majority of outlet with pair views.
- Suppose true signal is given with probability p=0.51.
- If media outlet gives wrong signal at round 0.
- Then at round 1 a fraction 1-0.51^2 of the pairs will have the wrong signal.
- This will remain so for all following rounds.

Questions

- We see that repeated majority can kill the aggregation of information.

- Are there any situations where it does not?

- Previous examples and follow up discussion from Mossel-Tamuz.

Questions

- We see that repeated majority can kill the aggregation of information.
- Are there any situations where it does not?
- Yes!
- Many examples …
- As usual democratic examples are good …
- For simplicity assume all degree are odd and a simple majority rule.
-

Questions

- We see that repeated majority can kill the aggregation of information.

- Are there any situations where it does not?

- Yes!

- Some easy examples:

- Suppose voters partitioned to many groups and in each group do repeated majority among all members of the group.

Questions

Some easy examples:

- Suppose voters partitioned to many groups and in each group do repeated majority among all members of the group.

- Each group decision is a monotone fair function so correct with probability at least p.

- If there are many groups by LLN we have aggregation.

Questions

Other easy examples:

- Suppose voters partitioned to many groups and in each group do repeated majority among all members of the group.

- There are only 5 groups of size n/5.

- In each group with prob → 1 get consensus to correct decision.

Questions

Are there more interesting examples?

- Information bounded.

- But can still flow around the network?

Questions

- As usual democratic examples are good …

- For simplicity assume all degree are odd and a simple majority rule.

- Let G be the directed graph where u→v means u takes v as part of its majority rule.
- We say that G is transitive if there exists a group \Gamma action on G which maps edges to edges.

- <u>Claim:</u> (M-Tamuz): If G is transitive then we have aggregation of information.
- <u>Proof:</u> Overall aggregation function is fair, monotone and transitive!

Questions

- Claim: (M-Tamuz) If G is transitive then we have aggregation of information.

- In fact a much weaker condition suffice.
- Let Γ be a group acting on G=(V,E).
- The orbits ofΓa are the sets $\Gamma a\ v\$$ for $\sqsubseteq\in\backslash$ V.
- Let m=minimal size of an orbit.

- Claim: (M-Tamuz): Γ acts on G then we have aggregation at p=0.5 + 1/log m.

- Moral: It is fine to have newspapers as long as there are many!

HW 1 - Plurality

- Let n = # voters, k = # of alternatives.
- Prove that for all n and k every plurality function is a monotone.
- Prove that for all n and k there exist a fair plurality function.
- Prove that for all n and k there exist a transitive plurality function.
- Prove that if n>1 and k>2 there exist no fair transitive plurality function.

HW 2 – Borda Rule

- Give two different distributions of signals which result in the Borda rule being an optimal voting rule.

HW 3 – Aggregating permutations

- Consider an aggregation model of permutations over k alternatives with a uniform prior over rankings and where:
- Each voter either receives the correct ranking with prob 1/k
- Or receives a ranking that is obtained from it by flipping the order of a single pair of adjacent alternatives.
- Define an efficient algorithm to estimate the true order given sampled signals.

HW 4 - The Gaussian Learning Model

- Consider the Gaussian learning model:

- Let G be a path with 6 nodes and assume that the original signals are: 0,0.2,0.4,0.6,0.8,1. Describe all of the declarations in the process and what it converges to.

- Find a graph where the number of iterations until convergence is greater than the diameter.

- (Open problem): Is there a family of graph where the number of iterations needed i(n) satisfies $i(n)/d(n) \rightarrow \infty$ where d(n) is the diameter of the n'th graph.

HW 5 - Cheaters on the de-Groot model

- Consider the de-Groot model on a weighted graph corresponding to a Markov chain.

- Suppose m out of the n players are cheaters. A cheater v repeatedly declares x(v) over and over again.

- Show that all players declarations converge to the same value.

- Write a system of linear equations for the resulting signal in terms of the cheaters declarations.

- Conclude that the non-cheaters declarations have no effect on the final signal.

HW 6 – Cheaters on Iterated Majority Models

- Consider the iterative majority model on (0.75 k, a/k) expanders and assume at least 1- a/(2k) receive the correct signal and there are at most ε a/(2k) cheaters.

- Derive bounds on the fraction f(t,ε) of vertices who hold the correct opinion at each iteration (f(t,ε) should not depend on n).

- Show that lim f(t,ε) $\rightarrow$ 1 as t $\rightarrow$ ∞ and ε $\rightarrow$ 0.