UC Berkeley
Department of Statistics

STAT 210B Mathematical Statistics

Problem Set 2
Spring 2015

Issued: Tues, Feb 3, 2015 **Due:** Thurs, Feb 12, 2015

Reading: For this problem set: Chapters 2 and 4, posted on website.

Problem 2.1
As defined in class, the Gaussian and Rademacher complexities of a compact set $\mathbb{A} \subset \mathbb{R}^n$ are given by

$$\mathcal{G}(\mathbb{A}) = \mathbb{E}\Big[\max_{a \in \mathbb{A}} \langle a, \, g\rangle\Big], \quad \text{and} \quad \mathcal{R}(\mathbb{A}) = \mathbb{E}\Big[\max_{a \in \mathbb{A}} \langle a, \, \varepsilon\rangle\Big],$$

where $g \in \mathbb{R}^n$ and $\varepsilon \in \mathbb{R}^n$ are i.i.d. vectors of standard Gaussian and Rademacher variables, respectively.

(a) Show that $\mathcal{R}(\mathbb{A}) \leq \sqrt{\frac{\pi}{2}}\,\mathcal{G}(\mathbb{A})$ for any set $\mathbb{A}$.

(b) Does an inequality of the same type hold with the roles of $\mathcal{G}$ and $\mathcal{R}$ reversed (and a different constant)? Justify your answer.

Problem 2.2
Recall the Euclidean ball $\mathbb{B}_2(1) = \{a \in \mathbb{R}^n \mid \|a\|_2 \leq 1\}$ and the ℓ_1-ball $\mathbb{B}_1(1) = \{a \in \mathbb{R}^n \mid \|a\|_1 \leq 1\}$.

(a) Show that there are constants $0 < c_\ell \leq c_u < \infty$ such that $\frac{\mathcal{G}(\mathbb{B}_2(1))}{\sqrt{n}} \in [c_\ell, c_u]$ for all n. (*Note:* We proved the upper bound in class, so really only the lower bound is left for you.)

(b) Show that there are constants $0 < c_\ell \leq c_u < \infty$ such that $\frac{\mathcal{G}(\mathbb{B}_1(1))}{\sqrt{\log n}} \in [c_\ell, c_u]$ for all n. (*Note:* We proved the upper bound in class, so really only the lower bound is left for you.)

Problem 2.3
Problem 4.1

Problem 2.4
Problem 4.4

Problem 2.5
Problem 4.8

Problem 2.6
Problem 4.9