Direct and Indirect Effects

This continues the discussion in section 5.5 of *Statistical Models*. Suppose we define a new hypothetical experiment, by the substitution of (17) into (18). This response schedule gives the "total effect" on Z of manipulating x:

$$\begin{aligned} Z_{i,x} &= c + dx + e(a + bx + \delta_i) + \epsilon_i \\ &= c + ae + (d + be)x + (e\delta_i + \epsilon_i). \end{aligned} \tag{19}$$

If we now substitute the exogenous X_i into (19), we get an ordinary regression equation,

$$Z_i = Z_{i,X_i} = c + ae + (d + be)X_i + (e\delta_i + \epsilon_i). \tag{20}$$

The error term in (20) depends on e, but this is harmless. The total effect $d + be$ of X on Z can be estimated in a regression of Z on X. This total effect is the sum of the "direct effect" d in (18), and the "indirect effect through Y," which is be.