OK... WHAT DO WE LEARN FROM THIS QUARTERLY REPORT GRAPH?... OTHER THAN WE PAY OUR BILLS LIKE CLOCKWORK ON THE FIRST OF EVERY MONTH!
Extending the Volatility Concept to Point Processes

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$2 \pi \neq 1$
Volatility sweeps global markets

US stock markets have dropped sharply, extending a global share sell-off amid fears about the effect of higher interest rates on the world economy.

There are concerns that higher rates will hit corporate profits and takeover deals, and dent consumer spending.

European markets were also jittery, with London's share index closing down for a fourth day and ending at its lowest level since the middle of March.

Analysts have warned that markets could remain volatile for a number of weeks.

"I think you've got bargain hunters out there for sure and I think you've got some people who are still scared," said Randy Frederic of Charles Schwab & Co.

"We're seeing the convergence of a whole host of sort of market drivers that are in play at the same time," he added.
Introduction

Is there a useful extension of the concept of volatility to point processes?

A number of data analyses will be presented
Why bother with an extension to point processes?

a) Perhaps will learn more about time series case
b) Pps are an interesting data type
c) Pps are building blocks
d) Volatility often considered risk measure for time series.

27 July Guardian.

“Down nearly 60 points at one stage, the FTSE recovered and put on the same amount again. But by the close it had slipped back, down 36.0 points.”

“inject billions into the banking system”
Volatility

When is something volatile?
When values shifting/changing a lot

Vague concept
Can be formalized in various ways
There are empirical formulas as well as models
Merrill Lynch.

“Volatility.

A measure of the fluctuation in the market price of the underlying security.

Mathematically, volatility is the annualized standard deviation of returns.

A - Low; B - Medium; and C - High.”
Financial time series.

\( P_t \) price at time \( t \)

“Return” data, \( Y_t = (P_t - P_{t-1})/P_{t-1} \)

*Empirical formula*

Realized volatility

mean\(\{|Y_s - Y_{s-1}|^p \mid s \text{ near } t\}\), \( p = 1 \) or \( 2 \)
Model based formula. GARCH

\[ Y_t = \mu_t + \sigma_t \varepsilon_t, \quad \varepsilon \text{ zero mean, unit variance, } t \text{ discrete} \]

\[ \sigma_t^2 = \alpha_0 + \sum \alpha_i [Y_{t-i} - \mu_{t-i}]^2 + \sum \beta_j \sigma_{t-j}^2 \quad \alpha's, \beta's > 0 \]

Volatility \( \sigma_t^2 \)

For \( \mu_s, \sigma_s \) smooth

\[ \text{mean}\{[Y_s - Y_{s-1}]^2 \mid s \text{ near } t\} \sim \sigma_t^2(\varepsilon_t - \varepsilon_{t-1})^2 \]
Crossings based.

\[
E\{[Y(t) - Y(t-h)]^2\} = 2[c(0) - c(h)] \approx -2c''(0)h^2
\]

Recognize as

\[
2c(0) \pi^2 [E(\#\{crossings of mean\})]^2
\]

for stationary normal

Consider

\[
s \#\{crossings of mean | near t\}
\]

as volatility measure

Weighted average of prices of 500 large companies in US stock market

Events

Great Crash Nov 1929

Asian Flu (Black Monday) Oct 1997

Zero crossings
S&P 500: realized volatility, model based, crossing based

Tsay (2002)

n = 792
Point process case.

locations along line: $t_1 < t_2 < t_3 < t_4 < \ldots$

$N(t) = \#\{t_j \leq t\}$

Intervals/interarrivals $X_j = t_{j+1} - t_j$

Stochastic point process.

Probabilities defined

Characteristics: rate, autointensity, covariance density, conditional intensity, \ldots

E.g. Poisson, doubly stochastic Poisson
0-1 valued time series.

\[ Z_t = 0 \text{ or } 1 \]

Realized volatility

\[ \text{ave}\{ [Z_s - Z_{s-1}]^2 | s \text{ near } t \} \]

Connection to zero crossings.

\[ Z_t = \text{sgn}(Y_t), \ \{Y_t\} \text{ ordinary t.s.} \]

\[ \Sigma [Z_s - Z_{s-1}]^2 = \#\{\text{zero crossings}\} \]
Connecting pp and 0-1 series. Algebra

\[ T_j = \langle \tau_j/h \rangle \quad <.> \text{ nearest integer, embed in 0's} \]

\[ h \text{ small enough so no ties} \]

\[ Y(t) = N(t+h) - N(t) = \int_{t}^{t+h} dN(u) \]

Stationary case \[ \int \]

\[ E\{Y(t)\} = p_N h \]

\[ \text{cov}\{Y(t+u),Y(t)\} = \int_{t+u}^{t+u+h} \int_{t}^{t+h} \text{cov}\{dN(r),dN(s)\} \]

\[ \sim p_N \delta\{u\} h + q_{NN}(u) h^2 \]

as \[ \text{cov}\{dN(r),dN(s)\} = [p_N \delta(r-s) + q_{NN}(r-s)] dr \, ds, \]

rate, \( p_N \), covariance density, \( q_{NN}(\cdot) \), Dirac delta, \( \delta(\cdot) \)
**Parametric models.**

*Bernoulli ARCH.* Cox (1970)

\[
\text{Prob}\{Z_t = 1|H_t\} = \pi_t
\]

\[
\text{logit}\ \pi_t = \sum \alpha_i Z_{t-i}
\]

\(H_t\) history before \(t\)

Fitting, assessment, prediction, … via \texttt{glm()}

*Bernoulli GARCH.*

\[
\text{logit}\ \pi_t = \sum \alpha_i Z_{t-i} + \sum \beta_j \text{logit}\ \pi_{t-j}
\]

Volatility \(\pi_t\) or \(\pi_t(1 - \pi_t)\)?
Convento do Carmo
“California” earthquakes magnitude $\geq 4$, 1969-2003

N=1805
Results of 0-1 analysis
P.p. analysis.

Rate as estimate of volatility

Consider \( \text{var}\{dN(t)\} \)

\[
\text{var}\{N(t)-N(t-h)\} \approx p_N h + q_{NN}(0) h^2
\]

Estimate of rate at time \( t \).

\[
\int k(t-u) dN(u) / \int k(t-u) du
\]

\( k(.) \) kernel

Variance, stationary case, \( k(.) \) narrow

\[
p_N \int k(t-u)^2 du / [\int k(t-u) du]^2 + q_{NN}(0)
\]
Example. Euro-USA exchange rate
Interval analysis.

$$X_j = t_j - t_{j-1}$$

Also stationary
California earthquakes

Cal quake intervals, mag >= 4

Realized volatility

garch(1,1) based

Crossings based
Risk analysis. Time series case.

Assets $Y_t$ and probability $p$

VaR is the $p$-th quantile

$$\text{Prob}\{Y_{t+1} - Y_t \leq \text{VaR}\} = p$$

left tail

If approximate distribution of $Y_{t+1} - Y_t$ by

$\text{Normal}(0, \sigma_t)$

volatility, $\sigma_t$, appears

Sometimes predictive model is built and fit to estimate VaR
Point process case.

Pulses arriving close together can damage

Number of oscillations to break object (Ang & Tang)

Suppose all points have the same value (mark), e.g. spike train.

Consider VaR of

\[ \text{Prob}\{N(t+u) - N(t) > \text{VaR}\} = 1-p \]

Righthand tail
Examples.

S&P500: $p = .05$ method of moments quantile

$$\text{VaR} = \$.0815$$

CA earthquakes: $u = 7$ days, $p = .95$ mom quantile

$$\text{VaR} = 28 \text{ events}$$
Case with seasonality – US Forest Fires 1970 - 2005

n=8481
Dashed line ~ seasonal
Example. Volatility, simulate Poisson, recover volatility

S & P 500 volatility


Linear process

Impulse response
Conclusion.

Returning to the question, “Why bother with extension?

a) Perhaps will learn more about time series case
b) Pps are an interesting data type
c) Pps are building blocks
d) Volatility often considered risk measure for time series.”

The volatility can be the basic phenomenon
Another question.

“Is there a useful extension of the concept of volatility to point processes?”

The running rate

gets at local behavior (prediction)
Some references.


Dettling, M. and Buhlmann, P. “Volatility and risk estimation with linear and nonlinear methods based on high frequency data”