Homework 2.

1. (Problem 2D.3) Let $\{u(t)\}$ be a (zero mean) stationary stochastic process with $R_u(\tau) = Eu(t)u(t-\tau)$ and let

$$\Phi_u(\omega) = \sum_{\tau=-\infty}^{\infty} R_u(\tau)e^{-i\tau\omega}$$

be its spectrum. Assume that

$$\sum_{1}^{\infty} |\tau R_u(\tau)| < \infty$$

Let

$$U_N(\omega) = \frac{1}{\sqrt{N}} \sum_{t=1}^{N} u(t)e^{-i\omega t}$$

Prove that

$$E|U_N(\omega)|^2 \to \Phi_u(\omega), \quad as\ N \to \infty$$

2. (Problem 2E.1) A stationary stochastic process has the spectrum

$$\Phi_v(\omega) = \frac{1.25 + cos\ \omega}{1.64 + 1.6cos\ \omega}$$

Describe v(t) as an ARMA process.