Volatility for Point and Marked Point Processes

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Introduction

Volatility has become an important topic for time series generally and for financial series particularly. The topic arises in point process problems also. Volatility is a vague concept, capable of being formalized in a variety of ways. A persistent notion is local variability. A definition from the financial world is: "A statistical measure of the dispersion of returns for a given security or market index. Volatility can either be measured by using the standard deviation or variance between returns from that same security or market index. Commonly, the higher the volatility, the riskier the security."

One can ask, what is the point of studying the topic? An important product of the study of volatility can be better assessment of risk. This will be discussed below. Studying volatility can also lead to better forecasting, and to checking that a process is under control.

Volatility measures are important in risk analyses involving ordinary time series, particularly many questions of risk analysis involve some form of variability. For example in insurance considerations the safety loaded pure risk premium can take the form

$$\lambda_1 P(t) + \lambda_2 \sigma(t) + \lambda_3 \sigma(t)^2$$

where the λ's are weights, $P(t)$ is the fair premium and $\sigma(t)$, or $\sigma(t)^2$, is the volatility at time t. The above expression is (10.2.6) in Daykin et al (1994).

Another measure of financial risk is the so-called Value at Risk (VAR). It has been defined as the maximum expected loss over a specified time period with a given confidence level. Losses in the time horizon will exceed the VAR only with prespecified probability α. Estimates of something like $\sigma(t)$ are required in its computation.

There will be more discussion of the time series case in the next section, but the motivating consideration of is a study of the concept of volatility in the point process case. It seems tied in with the issue of over and under-dispersion having the Poisson process as standard. One can ask if the local rate of a point process is varying. The definition of volatility could be the level. A basic issue is how to display point process data to bring out the presence or absence of volatility?

The subject matter of finance can be brought into the problem by imagining that there is a price of $1 associated with each event.

The layout of the paper involves discussion of the time series concept, and then the point process one. The material here is tutorial. A variety of empirical examples from various fields and some theory will be presented at the talk itself.

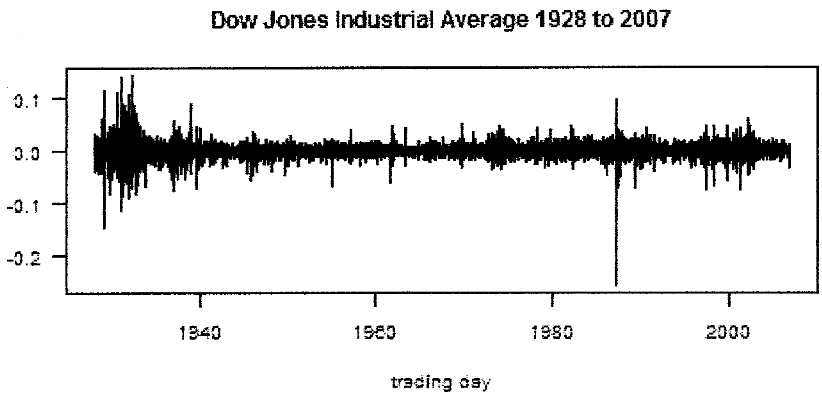
The Time Series Case

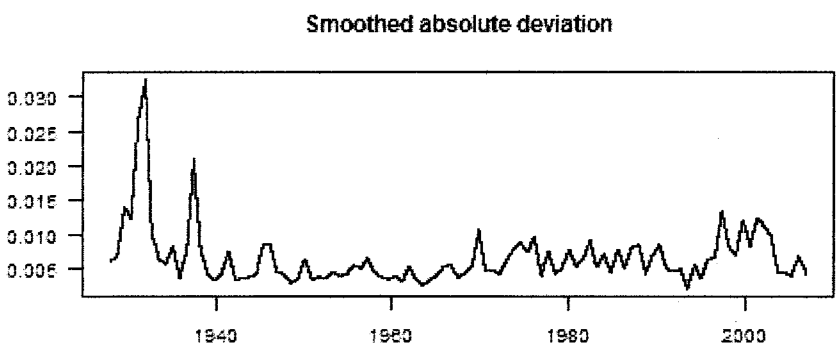
Let {Y(t), t=0, ±1, ±2, ...} denote a real-valued time series. One might consider the Dow Jones Industrial Average (DJIA) for example. It is based on the prices of some 30 stocks. In fact there is a stock, DIAMONDS (DIA) that can be purchased to mock the Index's level. The return, defined as R(t) = log Y(t)/Y(t-1), is studied for volatility.

Naïve estimates of volatility include:

$$\text{ave}\{\sum |Y(s) - Y(s\text{-}1)|\} \quad \text{and} \quad \text{ave}\{\sum [Y(s) - Y(s\text{-}1)]^2\}$$

The figure below plots the return and one naïve estimate of volatility.





One sees the volatility of the upper figure confirmed in the lower. There are seen to be intervals of high volatility corresponding to: the crash period of 1929, the Black Monday period of 1987, and the 9/11 period of 2001.

In a nonparametric formal approach one can consider

$$E\{[Y(t) - Y(t-h)]^2\} = 2[c(0) - c(h)] \approx -2c''(0)h^2$$

in the stationary case, where h is small. If one views the series $\{Y(t)\}$ as locally stationary, then with this definition volatility is getting at whether the second derivative at 0 of the autocovariance function changes much as one slides along the series.

Discussion of the topic in the literature often turns to GARCH models such as

$$R(t) = \mu(t) + \sigma(t)\varepsilon(t)$$

$\sigma(t)^2 = \alpha_0 + \sum \alpha_i\varepsilon(t-i)^2 + \sum \beta_j\sigma(t-j)^2$ with the α's and β's non-negative.

This is a is a parametric approach. Once the parameters are estimated an estimate of $\sigma(t)$ can be fed into risk computations.

The Point Process Case

The preceding material has been by way of easing into the point process case. Time is now continuous time and the process is on the real time line. One can denote a realization of the point process by a non-decreasing sequence, $\{\sigma_j\}$, or a non-decreasing function, N(t), counting the number of points up to and including time t, starting at time 0 say.

Point processes are a common data type and they are they are also building blocks of time series processes. There are both parametric and nonparametric forms of analysis.

Consider var$\{dN(t)\}$ as a measure of volatility of a point process. Writing dt = h, and supposing h small, one has

$$\mathrm{var}\{N(t)-N(t-h)\} = \iint[p_N\delta(r-s) + q_{NN}(r-s)]drds \approx p_N h + q_{NN}(0)h^2$$

with the process assumed stationary, having rate p_N and covariance density $q_{NN}(\cdot)$ and $\delta(\cdot)$ is the Dirac delta. Viewing the process as locally stationary the volatility is seen to relate to how near constant p_N and $q_{NN}(0)$ are. The Poisson variability shows itself in the ever presence of the term $p_N h$. The Poisson is the standard against which other processes are measured.

Depending on the sign of $q_{NN}(0)$ there can be over- or under-variation. It is direct to set down examples, theoretical and empirical, of each.

There is a time-series approach to working with point processes on the line. One considers the discrete time series of successive values $\sigma_j - \sigma_{j-1}$ i.e. the interarrival times. Interestingly if N(t) is stationary, so too is the series $\{\sigma_j\}$. One can look for volatility in each.

A parametric approach to the point process case has also been proposed by Engle and Russell (1988). The autoregressive conditional duration (ACD) model is a model for a point process in which the conditional intensity has the form

$$\lambda(t|H_t) = \lambda_0 ((t-t_{N(t)})/\psi_{N(t)+1})/\psi_{N(t)+1}$$

$$\psi_i = E(x_i \mid x_{i-1}, x_{i-2}, \dots , x_1) , x_i = t_i - t_{i-1}$$

and $x_i = \tau_i\varepsilon_i$ for some i.i.d. ε_i . λ_0 is a baseline. Estimation and prediction procedures have been developed.

Extensions

Various extensions are possible: the spatial case , the vector case, the marked point process, the long-tailed case and the integer-valued case. One might also consider random effect models.

Discussion and Summary

The work has provided some measures of local variability for point processes – both parametric and nonparametric. Locating volatile time periods may suggest explanatories, such as the 1929 stock market crash phenomena, or an understanding of a seasonal effect.

There is general discussion and empirical examples in Tsay (2002).

REFERENCES

Daykin, C. D., Pentikainen, T. and Pesonen, M. (1994). Practical Risk Theory for Actuaries. Chapman and Hall, London.

Engle and Russell (1988). Autoregressive Conditional Duration: A New Model for Irregularly Spaced Data, Econometrica

Tsay, R. S. (2002). .Analysis of Financial Time Series. J. Wiley, New York.

ABSTRACT

The intention is to propose and investigate several definitions for the concept of volatility in the case of point and marked point processes, for both the temporal and spatial domains. Volatility is a vague concept that can be, and has been, made precise in a number of ways. Point processes are basic to risk analysis, and volatility is often mentioned in work in that field so a useful extension of the concept can be anticipated cases are presented.. The Poisson case will provide a bench mark. Applications to financial statistics and other