STAT 150 HOMEWORK #3

SPRING 2024

Due Friday, Feb 9th, at 11:59 PM on Gradescope.

Note that there are *Exercises* and *Problems* in the textbook. Make sure you read the homework carefully to find the assigned question.

1. Pinsky and Karlin, Problem 3.8.2

2. Pinsky and Karlin, Problem 3.9.5

3. Pinsky and Karlin, Problem 3.9.8

4. Pinsky and Karlin, Problem 3.9.10

5. Let $(X_n)_{n=0}^{\infty}$ be a branching process with common distribution ξ having mean $\mu > 1$. Assume that $X_0 = 1$. Let $\phi_{\xi}(s) = \mathbb{E}[s^{\xi}]$ be the generating function of ξ. Recall that the extinction probability u_{∞} satisfies $u_{\infty} < 1$ in this case.
   (a) Explain why $\phi_{\xi}'(u_{\infty}) < 1$.
   (b) Let $u_n = \mathbb{P}(X_n = 0)$. Use part (a) to show that there exists $\rho < 1$ such that for all sufficiently large n,
   $$u_{\infty} - u_{n+1} \leq \rho(u_{\infty} - u_n).$$
   Hint: what is the formal definition of the derivative?
   (c) Show that there exist $b > 0, c < \infty$ such that for all n,
   $$\mathbb{P}(\text{ extinction } | X_n \neq 0) \leq ce^{-bn}.$$
   (As an exercise that you do not need to turn in, think of how you might interpret this inequality practically).