STAT 150 HOMEWORK #11

SPRING 2024

NOT DUE

1. Read Example 5.19 of Durrett.

2. Durrett 5.3 (Note: in part (a), Durrett means for you to prove that (X_n) is a martingale with respect to (X_n). In part (b), you should prove that (Y_n) is a martingale with respect to (X_n). In part (c), you will need to find an appropriate stopping time T.)

3. Durrett 5.4 (there is a typo in the statement of part (b): you should show that $(1/n) \log Y_n \to -1$). Hint: you will need to remember some calculus. The point of the exercise is that $M_n \to 0$ almost surely even though $\mathbb{E}[M_n] = 1$ for all n, hence the "unfair fair game". Of course, you know by now that expected value does not tell the complete story.

4. Durrett 5.6 (Hint: assume that $\sigma^2 > 0$ since otherwise the statement is trivial. What can you say about $\mathbb{P}(T < \infty)$?)