

## STAT 150 HOMEWORK #11

FALL 2023

### NOT DUE

1. Read Example 5.19 of Durrett.
2. Durrett 5.3 (Note: in part (a), Durrett means for you to prove that  $(X_n)$  is a martingale with respect to  $(X_n)$ . In part (b), you should prove that  $(Y_n)$  is a martingale with respect to  $(X_n)$ . In part (c), you will need to find an appropriate stopping time  $T$ .)
3. Durrett 5.4 (there is a typo in the statement of part (b): you should show that  $(1/n) \log Y_n \rightarrow -1$ ). Hint: you will need to remember some calculus. The point of the exercise is that  $M_n \rightarrow 0$  almost surely even though  $\mathbb{E}[M_n] = 1$  for all  $n$ , hence the “unfair fair game”. Of course, you know by now that expected value does not tell the complete story.
4. Durrett 5.6 (Hint: assume that  $\sigma^2 > 0$  since otherwise the statement is trivial. What can you say about  $\mathbb{P}(T < \infty)$ ?)