Stat153 Assignment 3 (due October 10, 2007)

1. **(Linear prediction)**
   Suppose that $\{X_t\}$ is an AR(1) process, we have observed X_1, X_3, and X_4, and we would like to estimate the missing value X_2. Find the best linear predictor of X_2 given X_1, X_3, and X_4.

2. **(Linear prediction)**
   Shumway and Stoffer problem 3.14.

3. **(ACF, PACF, and forecasting)**
   Consider the time series data in the file `sunspot.dat` on the website. It consists of $n = 285$ observations of the number of sunspots, from 1700 to 1984. This is a quantity that is believed to affect our weather patterns. This time series has been studied by many authors, including Yule (Philosophical Transactions of the Royal Society of London, Series A, 226:267–298, 1927) and Brillinger and Rosenblatt (Spectral Analysis of Time Series, B. Harris (Ed.), pp 153–188, Wiley, 1967).

   We will study the square root of the data (this transformation ensures that the variance is roughly constant). That is, for the series $Z_1, \ldots, Z_n$ from the file `sunspot.dat`, first compute the series $X_t = \sqrt{Z_t}$, and work with the series $\{X_t\}$ in what follows.

   (a) Compute the sample ACF and the sample PACF for this series.

   (b) By considering the sample ACF and sample PACF, decide which of the following would be appropriate for this data:
   AR(1), AR(2), MA(1) or MA(2).
   Use the data to estimate the parameters of the model that you choose.

   (c) Assuming that your fitted model is the true model, calculate forecasts X_{n+h}^n, for $h = 1, 2, 3, 4$. Calculate the 95% prediction intervals (assuming Gaussian noise).

   (d) The file `sunspot2.dat` on the website includes the number of sunspots for the years 1985 to 1988. Plot all of the data, and your forecasts and prediction intervals for the last four years. (Don't forget to undo the square root transformation by taking the square of your predictions.)

4. **(ARMA estimation)**
   Shumway and Stoffer problem 3.19.