Stat153 Assignment 2 (due September 26, 2007)

1. **(ACF of AR(1))**
   Shumway and Stoffer problem 3.2.

2. **(ACF of MA)**

   (a) Find the autocovariance function of the time series $X_t = W_t - 1.5W_{t-1} - W_{t-2}$, where $\{W_t\} \sim WN(0, 1)$.

   (b) Find the autocovariance function of the time series $X_t = \tilde{W}_t - 0.25\tilde{W}_{t-2}$, where $\{\tilde{W}_t\} \sim WN(0, 4)$. Compare with 2a.

   (c) Which of the MA models in 2a and 2b is invertible?

3. **(ARMA models)**
   For each of the following ARMA models, find the roots of the AR and MA polynomials, identify the values of p and q for which they are ARMA(p,q) (be careful of parameter redundancy), determine whether they are causal, and determine whether they are invertible. In each case, $\{W_t\} \sim WN(0, 1)$.

   (a) $X_t - 2X_{t-1} + 2X_{t-2} = W_t - \frac{1}{2}W_{t-1}$.

   (b) $X_t - 4X_{t-2} = W_t + 6W_{t-1} + 8W_{t-2}$.

   (c) $X_t + \frac{4}{3}X_{t-1} - \frac{4}{3}X_{t-2} = W_t + \frac{8}{3}W_{t-1} + \frac{4}{3}W_{t-2}$.

   (d) $X_t + \frac{1}{20}X_{t-1} - \frac{3}{5}X_{t-2} = W_t$.

   (e) $X_t = W_t - 0.4W_{t-1} + 0.04W_{t-2}$.

4. **(linear process representation of ARMA)**
   For those models of Question 3 that are causal, compute the first five coefficients $\psi_0, \psi_1, \ldots, \psi_4$ in the causal linear process representation $X_t = \sum_{j=0}^{\infty} \psi_j W_{t-j}$.

5. **(ACF of ARMA)**
   For those models of Question 3 that are causal,

   (a) Simulate 100 observations from each model. Compute and plot the sample ACF.

   (b) Compute the ACF.