

Stat153 Assignment 2 (due September 26, 2007)

1. (ACF of AR(1))

Shumway and Stoffer problem 3.2.

2. (ACF of MA)

- (a) Find the autocovariance function of the time series $X_t = W_t - 1.5W_{t-1} - W_{t-2}$, where $\{W_t\} \sim WN(0, 1)$.
- (b) Find the autocovariance function of the time series $X_t = \tilde{W}_t - 0.25\tilde{W}_{t-2}$, where $\{\tilde{W}_t\} \sim WN(0, 4)$. Compare with 2a.
- (c) Which of the MA models in 2a and 2b is invertible?

3. (ARMA models)

For each of the following ARMA models, find the roots of the AR and MA polynomials, identify the values of p and q for which they are ARMA(p,q) (be careful of parameter redundancy), determine whether they are causal, and determine whether they are invertible. In each case, $\{W_t\} \sim WN(0, 1)$.

- (a) $X_t - 2X_{t-1} + 2X_{t-2} = W_t - \frac{1}{2}W_{t-1}$.
- (b) $X_t - 4X_{t-2} = W_t + 6W_{t-1} + 8W_{t-2}$.
- (c) $X_t + \frac{4}{3}X_{t-1} - \frac{4}{3}X_{t-2} = W_t + \frac{8}{3}W_{t-1} + \frac{4}{3}W_{t-2}$.
- (d) $X_t + \frac{1}{20}X_{t-1} - \frac{3}{5}X_{t-2} = W_t$.
- (e) $X_t = W_t - 0.4W_{t-1} + 0.04W_{t-2}$.

4. (linear process representation of ARMA)

For those models of Question 3 that are causal, compute the first five coefficients $\psi_0, \psi_1, \dots, \psi_4$ in the causal linear process representation $X_t = \sum_{j=0}^{\infty} \psi_j W_{t-j}$.

5. (ACF of ARMA)

For those models of Question 3 that are causal,

- (a) Simulate 100 observations from each model. Compute and plot the sample ACF.
- (b) Compute the ACF.