Stat153 Assignment 1 (due September 10, 2007)

1. **(Weak versus strict stationarity)**
   This example shows that a stationary process is not necessarily strictly stationary.

   Suppose that $\{W_t\}$ and $\{Z_t\}$ are independent and identically distributed (i.i.d.) sequences, with $P(W_t = 0) = P(W_t = 1) = 1/2$ and $P(Z_t = -1) = P(Z_t = 1) = 1/2$. Define the time series model

$$
\begin{aligned}
X_{2t} &= W_t Z_t \\
X_{2t+1} &= (1 - W_t) Z_t.
\end{aligned}
$$

   Show that $\{X_t\}$ is stationary, but not strictly stationary.

2. **(Stationarity)**
   For each of the following, state if it is a stationary process. If so, give the mean and autocovariance functions. Here, $\{W_t\}$ is i.i.d. N(0,1).

   (a) $X_t = W_t + 2W_{t-1}$.

   (b) $X_t = W_1 + W_t$.

   (c) $X_t = W_t - t$.

   (d) $X_t = W_t W_{t-2}$.

   (e) $X_t = \begin{cases} W_1 W_t & \text{if } t > 1, \\ W_1 W_{t-1} & \text{if } t \leq 1. \end{cases}$

3. **(Stationarity and differences)**
   Shumway and Stoffer problem 1.8.

4. **(Computer exercise: AR processes)**
   Shumway and Stoffer problem 1.3.

5. **(Computer exercise: Sample ACFs)**
   Consider the moving average process

$$
X_t = \frac{W_t + W_{t-1}}{2},
$$

   where $\{W_t\} \sim WN(0, 1)$.

   (a) What are the mean and autocorrelation functions?

   (b) Generate $n = 100$ observations of this time series, and compute and plot the sample autocorrelation function.