Stat153 Assignment 1 (due September 10, 2010)

1. **(White noise)**
   We have seen that i.i.d. noise is white noise. 'This example shows that white noise is not necessarily i.i.d.

   Suppose that $\{W_t\}$ and $\{Z_t\}$ are independent and identically distributed (i.i.d.) sequences, with $P(W_t = 0) = P(W_t = 1) = 1/2$ and $P(Z_t = -1) = P(Z_t = 1) = 1/2$. Define the time series model

   $$X_t = W_t(1 - W_{t-1})Z_t.$$

   Show that $\{X_t\}$ is white but not i.i.d.

2. **(Stationarity)**
   For each of the following, state if it is a stationary process. If so, give the mean and autocovariance functions. Here, $\{W_t\}$ is i.i.d. N(0,1).

   (a) $X_t = W_t - W_{t-3}$.

   (b) $X_t = W_3$.

   (c) $X_t = t + W_3$.

   (d) $X_t = W_t^2$.

   (e) $X_t = W_t W_{t-2}$.

3. **(MA process and ACF)**
   Shumway and Stoffer problem 1.7.

4. **(ACF and forecasting)**
   Shumway and Stoffer problem 1.10a,b.
   (Notice that the autocorrelation function is denoted by ρ, not γ.)

5. **(Computer exercise: AR processes)**
   Shumway and Stoffer problem 1.3.

6. **(Computer exercise: Sample ACFs)**
   Generate $n = 100$ observations of the time series from Shumway and Stoffer problem 1.7:

   $$X_t = W_{t-1} + 2W_t + W_{t+1},$$

   where $\{W_t\} \sim WN(0, 1)$.
   Compute and plot the sample autocorrelation function.