Stat153 Assignment 6 (due Friday, November 18)

1. **(Discrete Fourier transform)**
   Show that, for any real sequence $x_1, \ldots, x_n$, the DFT is uniquely specified by its values at Fourier frequencies ν_j between 0 and 1/2, that is, the values of the DFT at frequencies ν_j between 1/2 and 1 are redundant.

2. **(Periodogram)**
   Generate four realizations of a Gaussian white noise process $w_1, \ldots, w_n$ for $n = 128$, 512, 1024, and 2048.

   (a) Compute and plot the periodogram in each case.

   (b) In each case, calculate approximate confidence intervals for $f(0.1)$, the spectral density for the time series at frequency 0.1.

   Explain your findings.

3. **(Smoothed Periodogram)**
   Consider the following smoothed spectral estimator.

$$\hat{f}(\nu) = \frac{1}{2\lfloor\sqrt{n}\rfloor + 1} \sum_{|j|<\sqrt{n}} I(\hat{\nu}^{(n)} + j/n),$$

   where $\hat{\nu}^{(n)}$ is the value i/n closest to ν. Repeat Question 2 using this smoothed periodogram in place of the periodogram.