Stat153 Assignment 5 (due Tuesday, November 15, at the lecture)

1. We'll prove that, for a polynomial ψ with real coefficients, $\psi(z)\psi(\bar{z}) = |\psi(z)|^2$.

   (a) Show that, for any complex z, $z\bar{z} = |z|^2$. Draw a sketch in the complex plane to illustrate.

   (b) Show that, for any complex z and any positive integer j, the complex conjugate of z^j is $\bar{z}^j$.

   (c) Hence show that, for any polynomial $p(z) = \sum_{j=1}^{k} a_j z^j$ with real coefficients a_j,

$$p(z)p(\bar{z}) = |p(z)|^2.$$

2. **(Rational spectral densities)**
   Calculate the spectral density for the following time series models. In both cases, show where the poles and zeros are in the complex plane, and explain how these affect the spectral density.

   (a) X_t, where $(1 + 0.95B^2)X_t = W_t$ and W_t is $WN(0,1)$.

   (b) Y_t, where $(1 - 0.09975B + 0.9975B^2)Y_t = (1 + 0.6B)X_t$ and X_t is as defined above.

3. **(Linear filters)**
   Let $\psi(B)$ be such that $Y_t = \psi(B)X_t$ for the time series defined in Question 2b above. For this linear filter, plot the function $z \mapsto |\psi(z)|^2$ defined on the complex plane.