Stat153 Assignment 2 (due September 23, 2005)

[Revised version: Sat Sep 17 20:36:28 PDT 2005]

1. **(Linear prediction)**
   Shumway and Stoffer problem 1.11.

2. **(ACF)**
   Let $\{X_t\}, \{Y_t\}$ be uncorrelated stationary sequences (that is, for all s, t, $\mathsf{Cov}(X_t, Y_s) = 0$). Show that $X_t + Y_t$ is stationary with autocovariance function equal to the sum of the autocovariances of $\{X_t\}$ and $\{Y_t\}$.

3. **(ACF of MA(1))**
   Shumway and Stoffer problem 2.1.

4. **(ACF of MA)**
   (a) Find the autocovariance function of the time series $X_t = W_t + 0.3W_{t-1} - 0.4W_{t-2}$, where $\{W_t\} \sim WN(0, 1)$.
   (b) Find the autocovariance function of the time series $X_t = \tilde{W}_t - 1.2\tilde{W}_{t-1} - 1.6\tilde{W}_{t-2}$, where $\{\tilde{W}_t\} \sim WN(0, 0.25)$. Compare with 4a.
   (c) Which of the MA models in 4a and 4b is invertible?

5. **(ARMA models)**
   For each of the following ARMA models, identify the values of p and q for which they are ARMA(p,q) (be careful of parameter redundancy), determine whether they are causal, and determine whether they are invertible. In each case, $\{W_t\} \sim WN(0, 1)$.
   (a) $X_t - 4X_{t-2} = W_t - 4W_{t-1} + 4W_{t-2}$.
   (b) $X_t - X_{t-1} + 0.5X_{t-2} = W_t - 0.2W_{t-1}$.
   (c) $X_t + 0.2X_{t-1} - 0.48X_{t-2} = W_t$.
   (d) $X_t + 1.9X_{t-1} + 0.88X_{t-2} = W_t + 0.2W_{t-1} + 0.7W_{t-2}$.
   (e) $X_t + 1.6X_{t-1} = W_t - 0.4W_{t-1} + 0.04W_{t-2}$.

6. **(linear process representation of ARMA)**
   For those models of Question 5 that are causal, compute the first five coefficients $\psi_0, \psi_1, \ldots, \psi_4$ in the causal linear process representation $X_t = \sum_{j=0}^{\infty} \psi_j W_{t-j}$.

7. **(ACF of ARMA)**
   For those models of Question 5 that are causal,
   (a) Simulate 100 observations from each model. Compute and plot the sample ACF.
   (Hint: Consider the following approach to simulating 100 observations from an AR(1) process: Generate an i.i.d. N(0,1) sequence, $w_1, \ldots, w_{100}$. Set x_0=0, and $x_t = \phi_1 x_{t-1} + w_t$ for $i = 1, \ldots, 100$. Then $x_1, \ldots, x_{100}$ is not stationary (because, for example, $\mathsf{Var}(x_1) \neq \mathsf{Var}(x_2)$ unless $\phi_1 = 0$). But if $|\phi_1| < 1$, the autocovariance values $\mathsf{Cov}(x_t, x_{t+h})$ approach the right values exponentially quickly with t. (See Problem 2.2.) The same statement is true for any causal ARMA process. Thus, one approach to simulating 100 observations from an ARMA process is to generate a sequence $x_1, \ldots, x_{T+100}$ in the manner described above, and discard the first T observations.)
   (b) Compute the ACF.