Stat 135 Fall 06 **A. Adhikari**

Exercises in Expectation and Variance

All the notation below will be familiar to you except perhaps $Corr(X,Y)$ which is the correlation between X and Y. Recall the definition of correlation:

$$Corr(X,Y) = \frac{Cov(X,Y)}{SE(X)SE(Y)} = E(X^*Y^*)$$

where X^* is X in standard units and Y^* is Y in standard units. You proved in an earlier homework that the correlation is a number in the range $[-1, 1]$.

1. The random variable X has expectation 20 and variance 5. The random variable Y has expectation 30 and variance 10. Also, $Corr(X,Y) = -0.8$.

a) Find the expectation and standard error of $X+Y$.

b) Find the expectation and standard error of $X-Y$. Compare the results with what you would have got had X and Y been independent with the same means and variances as in this problem. Give intuitive justifications for any differences that you see.

c) Find the expectation and standard error of $X-2Y+3$.

d) Find $Corr(X-2Y+3, 3X+Y-1)$.

e) Find $Corr(X+3, 3X+1)$.

2. A die is rolled twice. Let R_1 be the number of spots on the first roll and R_2 the number of spots on the second roll. Let $S = R_1 + R_2$ and let $D = R_1 - R_2$.

a) Two variables are said to be *uncorrelated* if the correlation between them is 0. Are S and D uncorrelated?

b) Are S and D independent?

3. A die is rolled 20 times. Let X be the number of 1's in the first 10 rolls, Y the number of 6's in the first 10 rolls, Z the number of 6's in the last 10 rolls, and W the number of 1's all 20 rolls.

a) Find the expectation and SE of each of X and Y. Without calculation, say what you think the sign of $Corr(X,Y)$ must be; justify your answer heuristically. Then find $Corr(X,Y)$.

b) Find the expectation and SE of each of X. and Z, and find $Corr(X, Z)$.

c) Find the expectation and SE of each of X and W, and find $Corr(X, W)$.

4. A standard deck consists of 52 cards, of which 4 are kings and 4 are queens. A bridge hand consists of 13 cards drawn at random without replacement from a deck.

Four bridge hands are dealt, thus using up the whole deck. Let X be the number of kings in the first hand, Y the number of queens in the first hand, and Z the number of queens in the fourth hand.

a) Find the expectation and SE of each of X and Y. Without calculation, say what you think the sign of $Corr(X, Y)$ must be; justify your answer heurisitically. Then find $Corr(X, Y)$.

Compare the calculations in this problem with those in **3a**.

b) Find the expectation and SE of each of X and Z, and find $Corr(X, Z)$. Compare the calculations with those in **3b**.

5. A population consists of N individuals who have an average income of μ with an SD of σ. Let n be an integer in the range $(0, N/2)$, and suppose $2n$ people are drawn at random from the population. Let X be the average income of the first n people drawn, and Y be the average income of the remaining n people drawn.

Find $Corr(X, Y)$, $E(X - Y)$, and $SE(X - Y)$ in terms μ, σ, n and N,

a) if the draws are made with replacement.

b) if the draws are made without replacement.

6 (continuing 5). It is typical that income distributions have long right hand tails, so let's assume such a shape for the distribution of incomes in the population.

Suppose that N is very large, e.g. in the hundreds of thousands. And suppose that n is large as well but small in relation to N, e.g. a few thousand.

If possible, give a clear description of the joint distribution of X and Y (with justifications for your answer),

a) if the draws are made with replacement.

b) if the draws are made without replacement.

Make sure that your answers to **5** and **6** are consistent.

7. Repeat Problem 6, now assuming that $n = N/2$. Yes, you can assume that N is even.

[If you have doubts about your answers, use some initiative – simulate!]