Stat 134, Fall 2008 **A. Adhikari**

Answers to "Variations on a Binomial Theme"

1. Let A be the event "more heads than tails", B the event "more tails than heads", and C the event "equal number of heads and tails". Then $A \bigcup B \bigcup C$ is a partition of Ω. That is, the three events are mutually exclusive and their union is the whole space. So $1 = P(A) + P(B) + P(C)$. Now use the fact that you're tossing a fair coin. Now $P(A) = P(B)$ by symmetry since the coin is fair. And $P(C)$ is the chance of 10 heads in 20 tosses, which is 0.1762 by the binomial formula with $n = 20$, $p = 0.5$, and $k = 10$. So $P(A) \ = \ \frac{1}{2}(1 - 0.1762) \ = \ 0.4119$.

2. "It takes more than 15 rolls to get 3 sixes" is the same as "in the first 15 rolls there are at most 2 sixes". This chance can be found by adding the binomial terms for $n = 15$, $p = 1/6$, and $k = 0, 1, 2$. The answer is 0.5322.

3. Since there are 10 tosses, "at least 4 heads and at least 4 tails" is just a roundabout way of saying "4 or 5 or 6 heads". The chance can be found by adding the binomial terms for $n = 10$, $p = 0.5$, and $k = 4, 5, 6$. The answer is 0.65625.

No, you can't just multiply *P(*at least 4 heads*)* by *P(*at least 4 tails*)*. Both of those events are based on the same 10 tosses and are not independent. It is correct to multiply *P(*at least 4 heads*)* by *P(*at least 4 tails | at least 4 heads *)*. But the second term is not easy to find unless you use the division rule, which will involve the probability of the intersection, which is what you were looking for in the first place.

4. This chance is 100%. The event must occur - in 10 tosses you can't have fewer than 4 of both kinds. You *didn't* start to do a calculation ... did you?!!

5. This is not directly binomial because you are looking at "red" and "blue", not "red" and "other" ("other" could be blue or green). But the method we used to derive the binomial formula works perfectly, as follows.

The chance that the first two tickets are red and the last three are blue is $0.3^2 \times 0.6^3$. But that's just one way the event could occur. You have to account for all the other ways. Each of those ways will have the same chance (two factors of 0.3 and three of 0.6). And there are $\binom{5}{2}$ different ways – the number of ways of putting 2 reds in 5 places with the rest of the places filled with blues. So the answer is

$$\binom{5}{2} 0.3^2 0.6^3 \ = \ 0.1944$$

6. The event in question is the union of two disjoint events A and B where:
$A =$ "left pocket found empty first, and k matches remain in right pocket"
$B =$ "right pocket found empty first, and k matches remain in left pocket"
Of course A and B have the same chance, so our answer is $2P(A)$.

To find $P(A)$ observe that for A to occur, $n + (n - k)$ matches have to be removed in all, with n of them coming from the left pocket; and on the next toss the left pocket has to be chosen. So there must first be $n + (n - k)$ tosses which result in exactly n heads, and then the next toss must be a head. This means that

$$P(A) \ = \ [\binom{2n-k}{n} 0.5^n 0.5^{n-k}] \cdot 0.5$$

and therefore the answer to the problem is

$$2P(A) \ = \ \binom{2n-k}{n} 0.5^n 0.5^{n-k} \ = \ \binom{2n-k}{n} 0.5^{2n-k}$$