

## Variations on a Binomial Theme

1. A coin is tossed repeatedly. Find the chance of getting more heads than tails in the first 20 tosses.

[The answer is not  $1/2$ . Nor does it require adding lots of binomial terms.]

2. I will roll a die until I have seen 3 sixes. Find the chance that I have to roll more than 15 times.

3. A coin will be tossed 10 times. Find the chance of getting at least 4 heads and at least 4 tails.

4. A coin will be tossed 10 times. Find the chance of getting at least 4 heads or at least 4 tails.

5. A box contains 1 green ticket, 3 red tickets, and 6 blue tickets. Five tickets are drawn at random with replacement. Find the chance that two of the tickets drawn are red and the rest are blue. Justify your answer.

[This is not a direct application of the binomial formula (why not?), but you should be able to do it if you understood the derivation of the formula.]

**6. Banach's Matchbox Problem.** A mathematician has  $n$  matches in each of two pockets. Every time he needs a match he tosses a coin. If the coin lands heads he takes a match from the left pocket, and if it lands tails he takes a match from the right pocket. What is the chance that when he first reaches into a pocket and finds no matches in it, there are  $k$  matches left in the other pocket?

[A clarification: The mathematician is not keeping track of how many matches he has used, and there's all kinds of junk in his pockets. So when he draws the last match out of a pocket he doesn't realize that there are no more matches in it. He has to reach into the pocket again to discover that the matches are all gone.]