Stat 134, Fall 2008 **A. Adhikari**

Answers to additional problems, Week 1

1a) $p_0 + p_1$

b) $\sum_{i=2}^{n} p_i$, but it's usually quicker to use the much simpler form $1 - (p_0 + p_1)$.

c) $1 - p_0$ as in the second form of (b), because "at least one" is the complement of "none". This one comes in very handy.

2a) The complement is "no reds", which is the same as "all blue". So the answer is $1 - (\frac{m-1}{m})^n$

b) This is the chance that all n are red, so $(\frac{1}{m})^n$.

c) The complement is simpler – it's "all red or all blue". So the answer is $1 - [(\frac{1}{m})^n + (\frac{m-1}{m})^n]$.

3a) The probability that the student has read both the plays is at least $\underline{0\%}$ and at most $\underline{5\%}$. Draw some Venn diagrams. The chance you want is the size of the intersection of the two groups, which may be empty if the groups are disjoint, and can be no larger than the size of the smaller group.

b) The probability that the student has read at least one of the plays at least $\underline{20\%}$ and at most $\underline{25\%}$. Draw some Venn diagrams, as in (a) only now you're looking at the union.

4a) Here are two ways of looking at this, with both equivalent answers given below. Either you could reason that the first draw has to yield one of the six letters, the second draw has to yield one of the remaining 5, and so on. Or you could note that the letters must come in the order O-R-A-N-G-E or in any of the other ways in which those six letters can be permuted; There are 6! such permutations and they each have the same chance. Answer:

$$\frac{6}{26} \cdot \frac{5}{25} \cdot \frac{4}{24} \cdot \frac{3}{23} \cdot \frac{2}{22} \cdot \frac{1}{21} \quad = \quad 6! \cdot \frac{1}{26} \cdot \frac{1}{25} \cdot \frac{1}{24} \cdot \frac{1}{23} \cdot \frac{1}{22} \cdot \frac{1}{21}$$

b) 1/26 by symmetry. Or you could do a calculation. One way is to note that there are 26! permutations of the alphabet, all equally likely. And 25! of them put the letter A in place 6. The ratio is 1/26.

5. In the large group let the proportion of respondents who would truthfully answer "yes" be p. That's what you are trying to estimate.

Assuming as usual that the result of a coin toss doesn't depend on who is tossing it, the expected proportion who will truthfully answer "yes" among those who get a head on the first toss is still p. Of those who get tails on the first toss, the expected proportion who will answer "yes" is 1/2, because their answer depends only on another toss.

This sets up probabilities along a tree whose first level is the H/T outcome of the first toss and whose second level is the yes/no answer given. You would expect that

$$0.4 \;=\; \text{P(answer yes)} \;=\; \text{P(heads and yes)} \;+\; \text{P(tails and yes)} \;=\; \frac{1}{2} \cdot p \;+\; \frac{1}{2} \cdot \frac{1}{2}$$

at least roughly. So your estimate of p would be 30%.