

Additional problems, Week 1

1. An integer is picked at random from $\{0, 1, 2, \dots, n\}$ according to the distribution $P(i) = p_i$ for $i = 0, 1, \dots, n$ with $\sum_{i=0}^n p_i = 1$. Find simple expressions for the probability that the integer picked is

- a) at most 1.
- b) more than 1.
- c) at least 1.

2. A balanced m -sided die will be rolled n times. One of the faces of the die is red and the rest are blue. Find the chance that

- a) the red face appears at least once.
- b) the blue faces never appear.
- c) both colors appear.

3. In a large class, 20% of the students have read *The Merchant of Venice* and 5% have read *Hamlet*. One student is picked at random. In parts (a) and (b) give the sharpest bounds that you can find.

- a) The probability that the student has read both the plays is at least _____% and at most _____%.
- b) The probability that the student has read at least one of the plays at least _____% and at most _____%.

4. Six letters will be picked one by one at random without replacement from the alphabet.

- a) Find the chance that the letters can be arranged to form the word ORANGE.
- b) Find the chance that the sixth letter picked is A.

5. In surveys about sensitive issues such as drug use people are often reluctant to give truthful answers to survey questions. So statisticians have devised survey methods which protect the respondents' privacy and still give the statisticians the estimates that they need. One of these methods is called "randomized response".

Here is one way to use randomized response. Suppose the survey question has a yes/no answer (e.g. "Do you use heroin?") and the goal is to estimate the percent of people whose answer is yes. The respondent is asked to toss a coin and answer truthfully if the coin lands heads. If the coin lands tails, the respondent tosses the coin again and answers "yes" if this second toss lands heads, "no" otherwise. All this happens out of sight of the surveyor. In the end the surveyor receives a yes/no response but has no way of knowing whether it is truthful.

Suppose you ran such a survey on a large group of respondents and got 40% "yes" responses overall. What would be your estimate of the percent of respondents who would truthfully answer "yes"? Justify your answer.