Stat 134, Fall 2008 **A. Adhikari**

Answers to questions on Gamma Densities page

Each of the densities below is defined on $(0, \infty)$. To see if it's a gamma density you have to check whether the functional form is

$$constant_1 \ variable^{power} \ e^{-constant_2 \cdot variable}$$

Once you know that it's a gamma density, to identify its parameters r and λ you have to re-write it as follows (I have used t for the variable but you can use anything you like, within reason):

$$constant \ t^{r-1}e^{-\lambda t}$$

The constant is

$$\frac{\lambda^r}{\Gamma(r)}$$

(i) $f(x) = (x^2e^{-x})/2$. Gamma density, $r = 3$, $\lambda = 1$. The theory says that the constant should be $1^3/\Gamma(3) = 1/(2!) = 1/2$, which agrees with what is given.

(ii) $g(u) =$ constant e^{-u}. Gamma density, $r = 1$, $\lambda = 1$, same as exponential $(\lambda = 1)$ density. The constant is 1.

(iii) $f(u) =$ constant e^{-u^2}. Not a gamma density. The exponential function involves the square of the variable. (It's not normal either, because it's defined on $(0, \infty)$. Can you identify it as the density of a function of a particular normal variable? Try. And find the constant.)

(iv) $f(t) =$ constant $e^{-t/2}/\sqrt{t}$. Gamma density, $r = 1/2$, $\lambda = 1/2$. This is the density of the square of a standard normal variable, as you will see in Section 4.4. The constant is $(1/2)^{1/2}/\Gamma(1/2)$ which $(1/2)^{1/2}/\sqrt{\pi}$ (Section 4.4 again).