Fermi Paradox: a simulation solution

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Introduction

The paradox was proposed by Fermi in 1950 during a lunch with Edward Teller, Hilbert York and Emil Konopinski. When they were talking about mundane topics, Fermi asked a question:"Where is everybody?"(Webb, 2002) His partners immediately understood what he was talking about: If the extraterrestrial civilization exists, why we have not met them till now? If we believe that there is nothing special about earth and solar system, there should be other civilizations in the universe as its enormous size containing countless probabilities. Under such assumptions, the paradox occurred. To provide a forceful explanation, many efforts have been paid on the analysis on *Drake equation*.

Drake equation:

$$N = R^* \cdot f_p \cdot n_e \cdot f_\ell \cdot f_i \cdot f_c \cdot L$$

The number of civilization in the universe is represented by *N*. To estimate it, we need to know the yearly rate *R* at which stars forms the galaxy; the probability f_p of stars that possess planets; the number n_e of planets with suitable environment for life; the probability f_l of suitable planets on which life actually develops; the probability f_i of these planets on which develops intelligence; the probability f_c of these intelligence that could communicate with other civilizations and the time *L* that such a culture would devote to communication (Webb, 2002). Though the equation is complicated as above, it can be just divided into 3 main factors.

Consider the following equation:

n=Npq

This equation is provided by Professor David Aldous, which can be viewed as a simple version of *Drake equation.* In this equation, *N* is the number of suitable planets for life; *p* is the chance that an intelligence species would develop a capability to communicate with others and *q* the chance that such a species would survive in such a way as to be observable (Aldous, 2010). Aldous (2010) has analyzed that *p* could be small, but this paper would concentrate on *q* given a predetermined *p* value.

In order to make the analysis easier, all the analysis would be based on the simulation results from an abstract model of the civilizations. The following would illustrate that the probability *q* is actually largely depend the expected life of civilizations, with longer expected life contributing to a larger *q*.

Model description

This model is referring to I.Bezsudnov and A.Snarskii's (2010) work, BS-model. The following model is built upon several assumptions. First, there is nothing special about time and position in the universe, which means that during each time step and at every position, a civilization could appear. Second, a civilization could expand its communicable scope by scientific and technology progress, but also could stay unchanged or even shrink the scope due to some catastrophe. Third, a civilization could die or disappear. The death of a civilization could be the extinction of this civilization and also could be the case that this civilization lose interest to communicate with others so as to limit their scope within their own planet. Last but not least, the interaction among civilizations would affect their existing status. They could contribute to each other's development but also could harm each other. Based on these criterions, the following model was proposed.

1. Preliminary setting

Project the whole universe onto a plane, denoting stars as some points on it. This plane could contain N points in it, which means the maximum amount of civilizations is N. On the universe timeline, we define one million years as one unit of time step and the whole simulation would process during T time step. And we also define the radius of solar system being one unit of distance in this model. Civilizations could be located at some points on this plane. For each civilization, there is a zone of radius r centered at the point of this civilization represent its communicable scope.

2. Initial condition and birth

The initial condition is an empty plane, meaning no civilization exists. During each step of time, new civilization may appear in the universe with probability p. For an individual civilization, its appearance would follow a Bernoulli distribution. Then for each step, the total number of new civilizations would follow a binomial distribution which can be approximated by normal distribution $N(Np, Np(1-p))$. For each new born civilization, it would be obsessed with following characters: expected life time, a random value between 0 and T_0; original scope 0, meaning that it initially only develop on its birth-planet; relative position, which is denoted by the distance with other already existed civilizations. Due to the restriction of the computer, the distance between any two civilizations cannot exceed 2^{63} and the distance between any two civilizations would be a random number between 0 and 2^{63}. Both the expected life time and the relative position would be simulated at the birth of a new civilization.

3. Growth and death

The growth and development of civilizations would depend on their existed time and expected life time. If the civilization exists less than expected life time, during each time step, its scope may expand with probability p_1 or being unchanged.

Similarly, if the existing time exceeds its expected life time, the civilization would shrink with probability p_2 and with probability $1-p_2$ the size would not change. For simplicity, we set the size of expansion or shrinkage just being 1 distance unit and also let $p_1 = p_2 = 0.5$. If a civilization shrinks its scope with radius 0, then it is considered to be dead.

4. Interaction among civilizations

   If, during the growth, one civilization meets another, meaning that the distance between two civilizations is less than the sum of their scope radiuses, they form a cluster and expected life time of each would be affected by bonus time T_b. With probability p_3, the encounter would offer bonus T_b time to each civilization and with probability $1-p_3$, each civilization would suffer $-T_b$ time punishment. Also, for simplicity, we set $p_3=0.5$. If a civilization A meet another one B while A is already in a cluster, B is considered to meet all the other civilizations in A's cluster. If civilization A and B and in the same cluster, then their encounter would assert no influence.

By a simple summary, we could find the following parameters in the model should be predetermined:

N: the maximum amount of civilizations in the plane

p: the probability of a new civilization appears

T: the total time step of simulation

T_0: the upper limit of expected life time

T_b: bonus time (punish time) during the interaction

In order to analyze the chance that a civilization that could be observable, we would fix N and p in the analysis. Considering the homogeneity of time in the model, the variation of total time step would also be ignored. The main focus of this report would be the effect of T_0 and T_b.

Simulation analysis

1. Preliminary setting

All the following results are set in the same initial conditions, with maximum amount of civilizations N being 1×10^8 and the probability of birth p being 1×10^{-8}, which means that during each time step, the expected number of newborn civilization is 1. The following analysis would mainly focus on two parts, the amount of civilizations and the civilization degree of the universe.

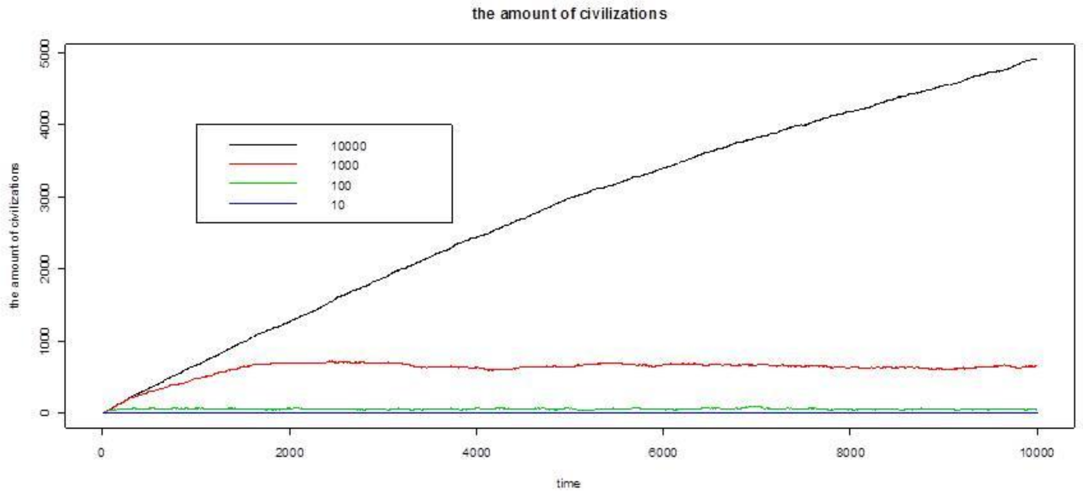
2. Analysis of the amount of civilizations

The amount of civilizations would demonstrate the primary information about this universe, which could demonstrate the effect of T_0 and T_b.

2.1 The effect of maximum expected life time T_0

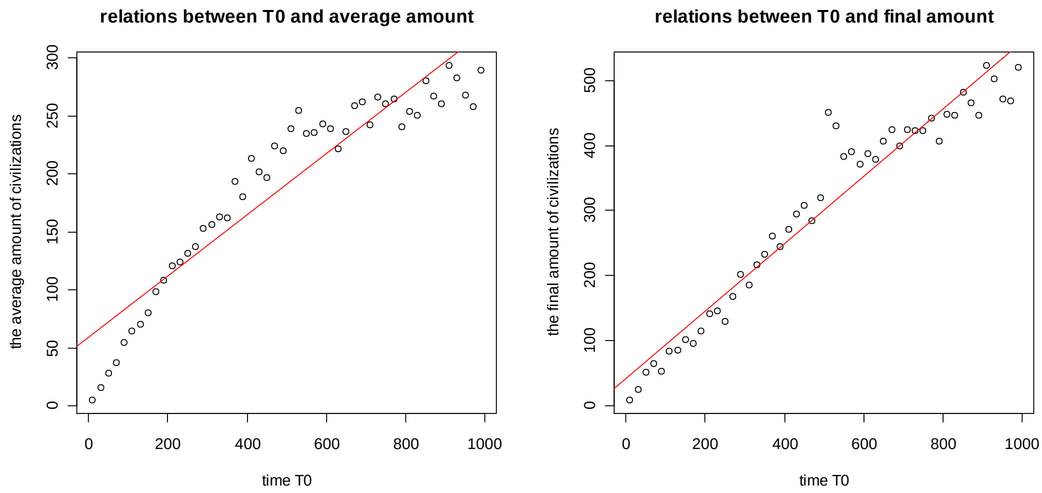
Similarly, putting T=10000, T_b=1, by altering the value of T_0, we can get the following picture. (Fig. 1)

Fig. 1



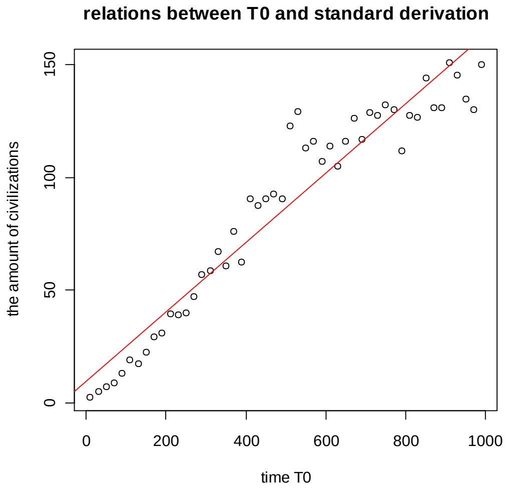
It can be easily observed that T_0 determines the amount of civilizations. Given a larger T_0, we would have a larger mean value as well as more civilizations in the final state. We can verify this result by linear regression analysis. These data are generated from T_0=10 to T_0=*1000*, under the condition of T=1000 and T_b=1. (Fig.2)

Fig. 2



From the result of linear regression, we can confirm such relation that T_0 is positively proportional to the amount. Similarly, the standard derivation of amount of civilizations is also positive proportional to T_0, which is shown in Fig. 3.

Fig. 3

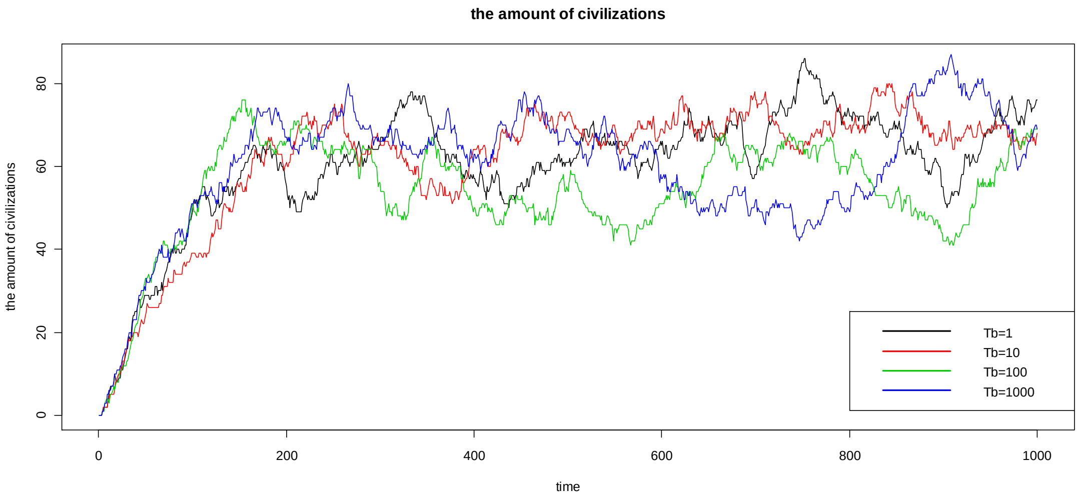


The above can be concluded that, at a certain universe, the amount of civilizations would be larger if T_0 could be longer. And at the same time, the variation of amount would also be larger, making it more unstable. Namely, with longer life expectancy, the universe would be obsessed with more civilizations while the amount during each time step may differ widely.

2.2 The effect of bonus (punish) time T_b

Setting T=1000 and T_0=100, we can observe the effect caused by T_b. (Fig. 4)

Fig. 4



By observation, T_b value does not cause much effect on the shape of amount, which needs more exploration by linear regression. (The following is the analysis result by program R as well as Fig.5)

Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) 55.769118 0.629905 88.536 < 2e-16 ***

y[, 1] 0.004047 0.001083 3.737 0.000313 ***

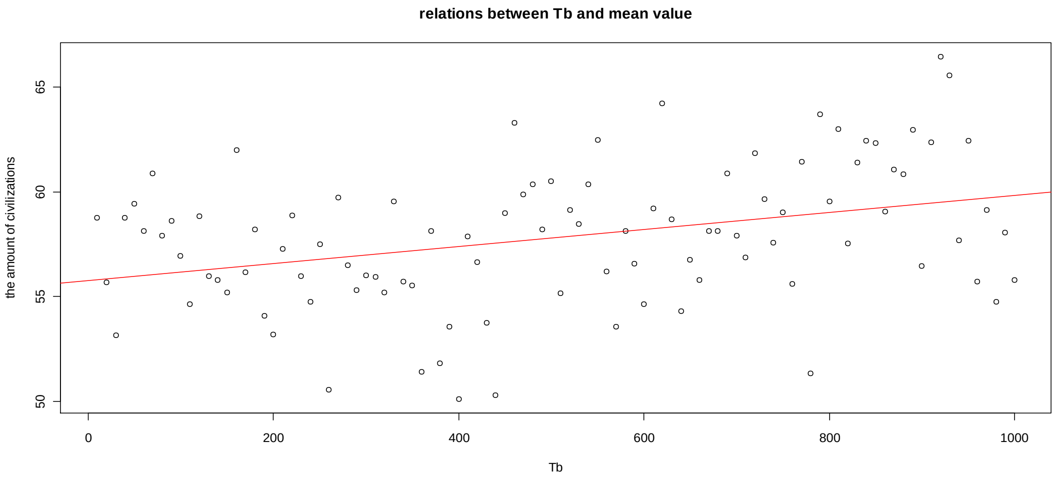
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.126 on 98 degrees of freedom

Multiple R-squared: 0.1247, Adjusted R-squared: 0.1158

F-statistic: 13.97 on 1 and 98 DF, p-value: 0.0003132

Fig.5



From the analysis result and the graph, we can roughly conclude that T_b contributes to the increasing of mean value, but only at a very slightly degree. Similarly, the result of standard derivation was generated as follows. (Fig. 6)

Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) 12.598730 0.299586 42.054 < 2e-16 ***

y[, 1] 0.001653 0.000515 3.209 0.00180 **

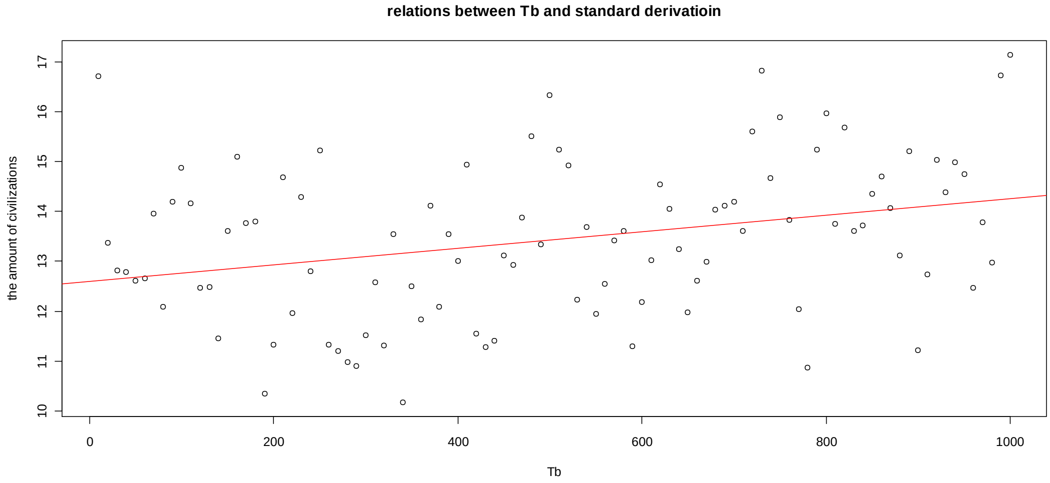
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.487 on 98 degrees of freedom

Multiple R-squared: 0.09507, Adjusted R-squared: 0.08583

F-statistic: 10.3 on 1 and 98 DF, p-value: 0.001803

Fig.6

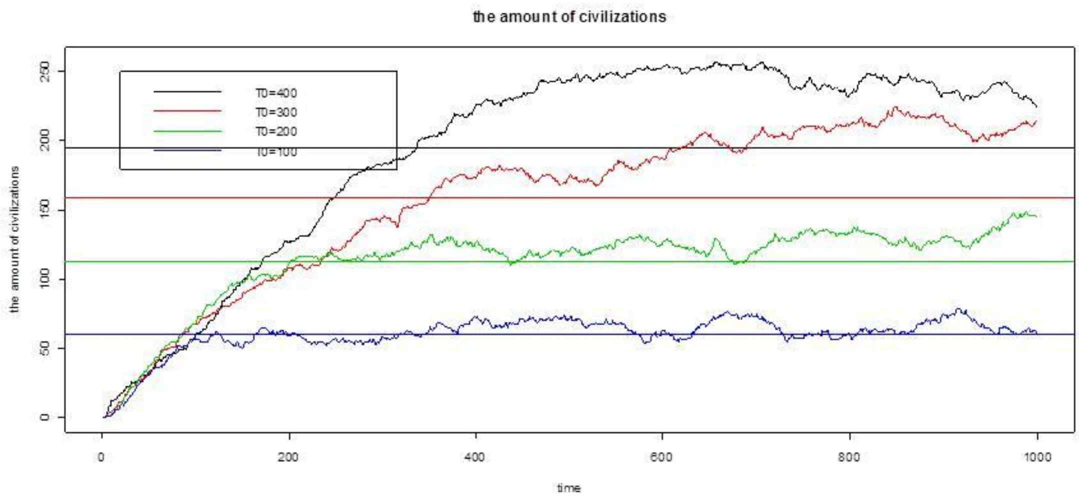


The result is very similar to the result of mean value, T_b contributing slightly to the increase of standard derivation. Above all, T_b only has minor effect on the behavior of amount of civilizations, though it contributes to the increasing of mean value and standard derivation. In other words, no matter how large the effect of interaction among civilizations, it would not largely affect the total amount of civilizations in this universe.

2.3 Shape

It is now acknowledged that T_0 is the main factor affecting the behavior of amount of civilizations, and the following analysis would focus on its effect by analyzing the shapes of graphs. Fig.7 was generated under the condition T=1000 and T_b=1, with different color depicting different T_0 and horizontal lines showing corresponding mean value.

Fig. 7



The behaviors of mean value lines are consistent with the analysis above, shifting up as T_0 increases. During the initial period of simulation, the amount would increase steadily, while the length of this period is related to T_0. With larger $T_{0,}$ the period would be longer. By observation, we can roughly set the initial period end at the first intersection between the amount curve and their corresponding mean value line. After the initial period, for small T_0, the amount of civilizations would fluctuate around mean value. However, for large T_0, the amount would keep growing with slower pace until reaching some steady state. By the analysis above, we can conclude that, during the initial period of universe, the amount of civilizations would increase steadily until reach some steady amount and after that the amount would fluctuate around this steady value. Meanwhile, both the length of "initial period" and the quantity of "steady amount" are affected by T_0. Given larger T_0, the initial period would be longer and the steady amount would be larger.

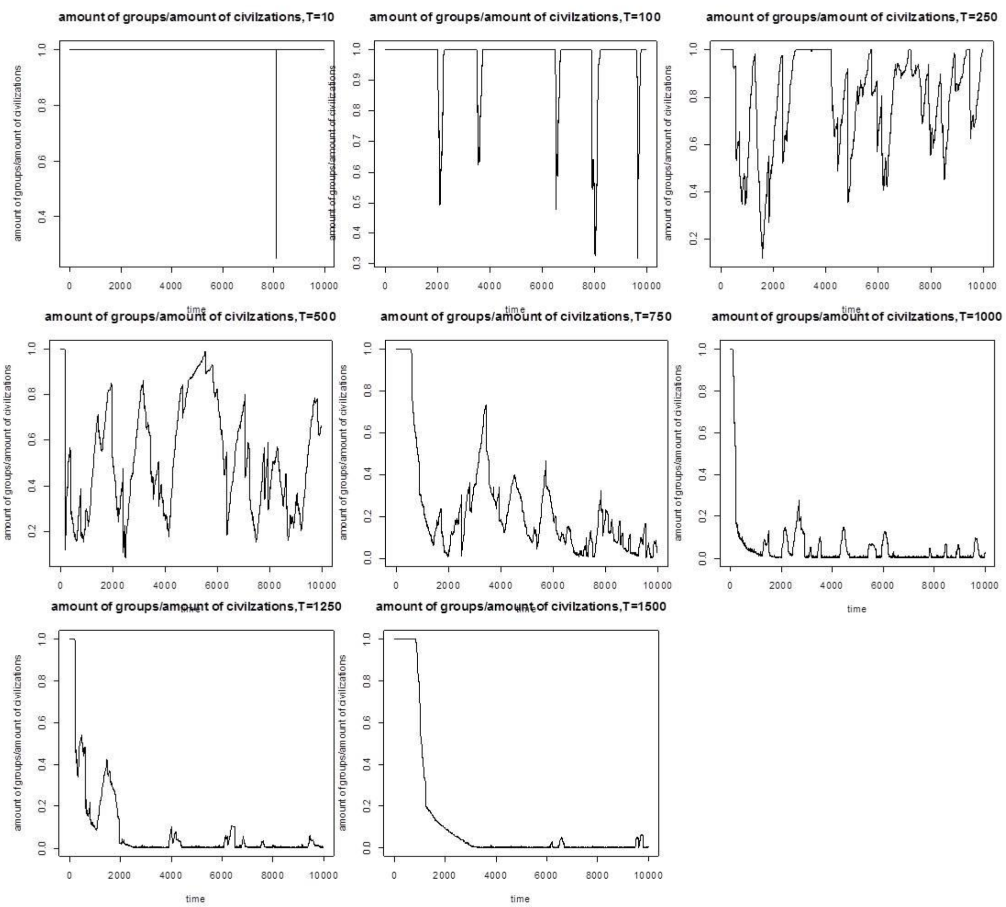
3. Analysis of the civilization degree of the universe

The civilization degree is measured by the ratio of clusters among total civilizations, defined as "the amount of clusters/ the amount of civilizations". The smaller ratio represents tighter connectedness between civilizations. Also define the ratio being 1 if there is no civilization in the universe. The case that the ratio curve converging to 0, would mean all civilizations in the universe are connected.

3.1 The effect of maximum expected life time T_0

In order to focus on the effect of T_0, we set T=10000 and T_b=1. By simulating different values of T_0, we could get following pictures. (Fig. 8)

Fig. 8



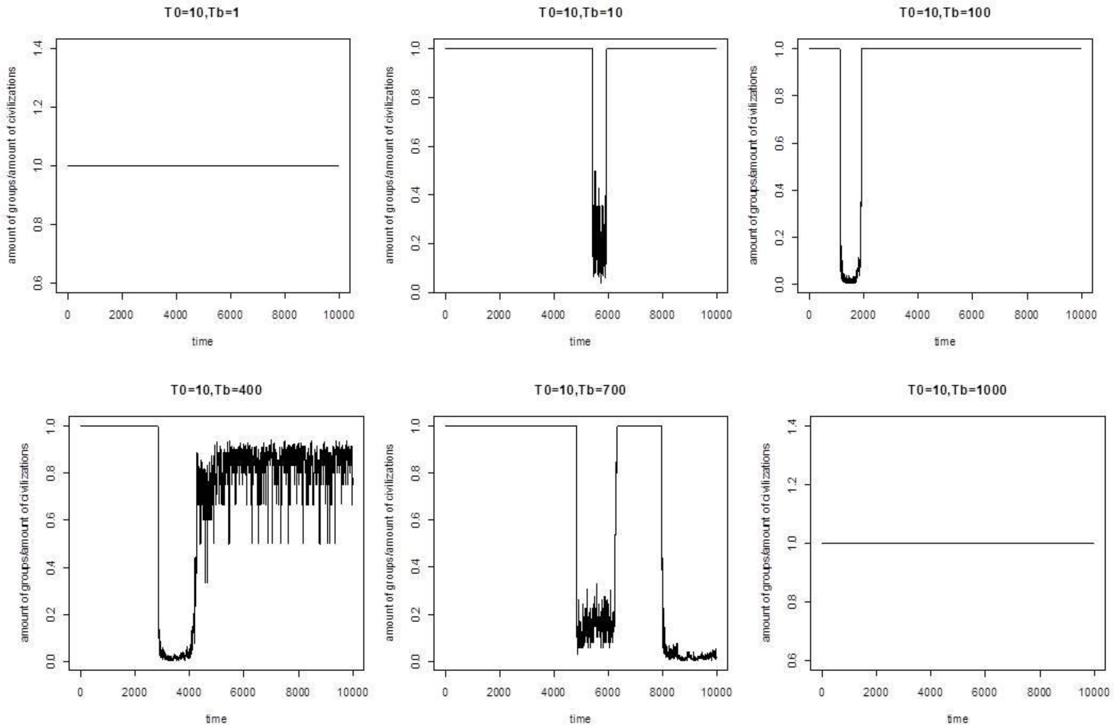
It can be observed from Fig.10 that with large T_0, the ratio would eventually converge to 0. When T_0 is very small, it would not converge, and most time the ratio maintains 1. When T_0 is larger, it would not converge either but the ratio would become extremely unstable. Considering fluctuation, it can be observed that the ratio curve fluctuate more violently as T_0 increases, while it would become stable again when T_0 is large enough. Such results imply the key factor determining the interacting status among civilizations is T_0. The civilizations in the universe would eventually know each other as long as T_0 is big enough. However, even T_0 is small, these large fluctuations shows that there are still possibility that at some time point when all civilization in the universe are connected.

3.2 The effect of bonus (punish) time T_b

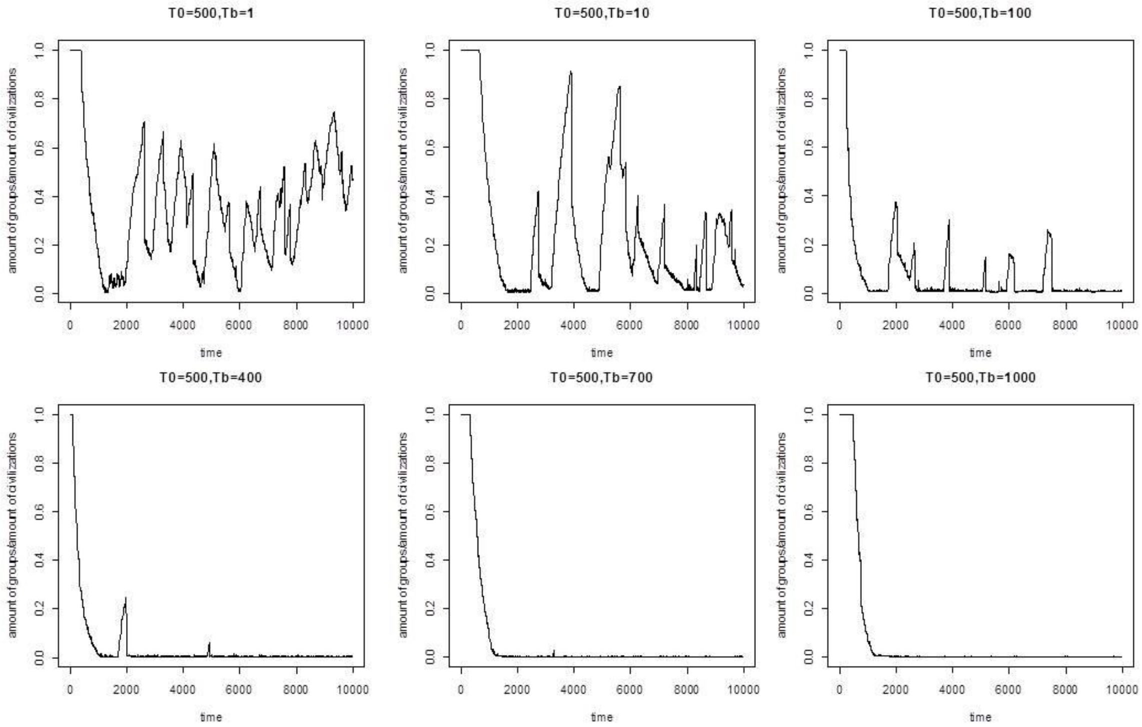
Similarly, in order to study T_b, we set T=10000. Considering the results above, we would test T_b under different T_0 values 10, 500 and 1500.

Fig. 9

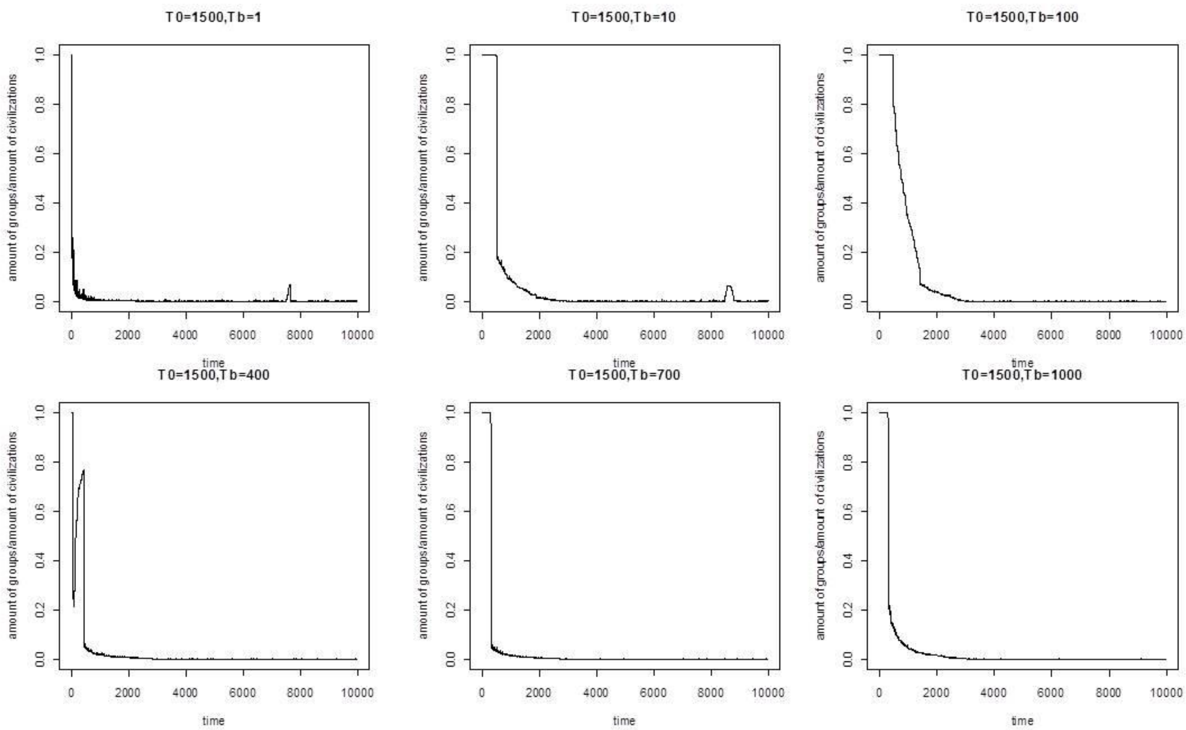
$T_0 = 10$



$T_0 = 500$



$T_0 = 1500$



It is interesting to notice that when T_0 is small, the effect of T_b is ambiguous. These results above have not shown a consistent result. Under some circumstance, it makes the curve more unstable; but under other circumstance, it makes the curve more stable. However, when T_0 become larger making the ratio fluctuates violently,

T_b becomes an important factor influencing the stability of the curve. Larger T_b would lead to less fluctuation and also contribute to the convergence of the curve. When T_0 is large enough, T_b's effect is also ambiguous, which have not shown evident effect on the curves.

Conclusion

From the analysis above, we can get the following conclusions:

1. T_0 is the main factor determining the behavior of civilizations.

   T_0 determines the amount of civilizations in the universe. Larger T_0 leads to larger amount of civilizations and larger T_0 would contribute to more violent fluctuation of the amount along the timeline. T_0 also determines the degree of civilization in the universe. For small T_0, the civilization degree would be low, which means each civilization would develop separately without knowing each other. If T_0 becomes a little larger, the civilization degree would behave very unstable, which depicts a peculiar scene that at one moment, most civilizations in the universe are connected but after a short period of time, they may lose contact with each other. However, when T_0 is large enough, the civilization degree would increase as time goes and keep a high level for the rest of time, which means that all the civilizations would be connected eventually.

2. T_b would assert its influence on some aspect.

   T_b would also contribute to the increasing of amount of civilizations, but only at a slight degree. The main effect of T_b would be shown on the civilization degree. When T_0 is such a value that makes the ratio fluctuates violently, large T_b would stabilize its behavior and help keep the ratio at a high level. Under this condition, given T_b large enough, all the civilizations would also be connected eventually.

From the conclusion above, it can be concluded that the chance that our human can interact with other civilizations actually depends on T_0. If T_0 is large enough, there would be large amount of civilizations existing in the universe and more importantly, they would be highly connected at the same time. Under this circumstance, we would eventually meet others as long as we can survive in the history of universe. Even if T_0 is not extremely large but still considerable large, we still have chance to meet others if T_b is large enough. In this case, the moment when we meet others may be different from the former case. Because the high civilization degree is closely related to T_b, which means that the interaction among civilizations would have a profound effect on each other, the encounter with other civilization would also generate a deep effect on us, which could be intense stimulation that helps us achieve an unbelievable prosperity or could also be a fierce catastrophe that erases our existence in the universe from that point on.

We would have a low chance to meet others except under these two cases above. In fact, both cases require long expected life time for us. In the first case, if and only if we can survive long enough that we can meet others. In the second case, if our expected life time is not long enough, our civilization may be destroyed by the encounter with others. The expected life time may depend on our level of science and technology; may depend on our level our social development and may also depend on our military might. The only possibility that we could meet others lies on the length of our survival time.

Acknowledgements

The model is still very rough to stimulate the behavior of civilizations, which could be improved. The scale of simulation could also be larger with a better computer. This report is inspired by I. Bezsudnov and A. Snarskii's, which helps the author form the original model.

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