

Research Proposal

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Simulation of Spatial Network Model of City Growth and Formation

1. Background and Motivation

The research is based on the paper “A Spatial Model of City Growth and Formation” <https://arxiv.org/abs/1209.5120>. In this paper, a model which discusses the influence of a city is introduced. In the model, the city populations grow at rates proportional to the area of their “sphere of influence”. Specifically, the influence of a city depends on its population and distance from other cities.

The primary motivation of this research is to do some simulation of the model. While analyzing the model, some problems which are hard to solve through a theoretical way will be discussed by simulation. Based on the model, problems related to real-world cities will be explored.

2. Goals

The research will mainly focus on the simulation of the model. The first task is to discuss the influence of the parameter in the model and use simulation to discuss some result derived from theoretical proof. In addition, the sensitivity of parameters will be discussed.

The primary task of this research is to discuss the problems which are difficult to solve theoretically. Based on Aldous’s technical notes on the rigorous proof of the model, there are several problems:

- **Coupling:** When the time $t+1$ immigrant arrive at a given position, there are two alternatives (join existing city or found new city). One might hope that the two conditioned future processes could be coupled in some manageable way (e.g. the second alternative differs from the first only by transferring some inhabitants of other cities to the new city). But rigorous coupling construction of this type is still unknown.
- **0-1 laws:** We do not know any general result saying that the asymptotic behavior of the process is unaffected by initial behavior; and indeed in certain settings we know 0-1 laws do not hold.
- **Repulsion of cities:** Because the influence function is infinite at a city, the positions of cities should intuitively be “less clustered” than random cities. But we don’t know any simple formalization of this idea.

Also, there are several special cases which are not discussed in a detailed way in the notes. The research will focus on these critical cases and explore some potential important properties. For example, the research will try to find the borders between cities and illustrate the formation of cities in an accurate way. Furthermore, some

related work such as the size and spatial distribution of real-world cities will be discussed.

Moreover, if I have extra time, I would like to try some mathematical proof related to this model.

3. Methodology

The simulation will be based on Matlab and C++. The key part is visualizing the model through some clear charts. Moreover, to make the project richer, I will try to draw moving pictures to illustrate the growth and formation of the cities. Theoretical proof will be utilized as a tool to verify the result of simulation. Specifically, the simulation will follow the heuristic analysis of the theoretical model.

Reference

Aldous's paper about the model: "A Spatial Model of City Growth and Formation"
<https://arxiv.org/abs/1209.5120>

Aldous's technical notes on the rigorous proof of the model
<https://www.stat.berkeley.edu/~aldous/Research/OP/cities-notes.pdf>