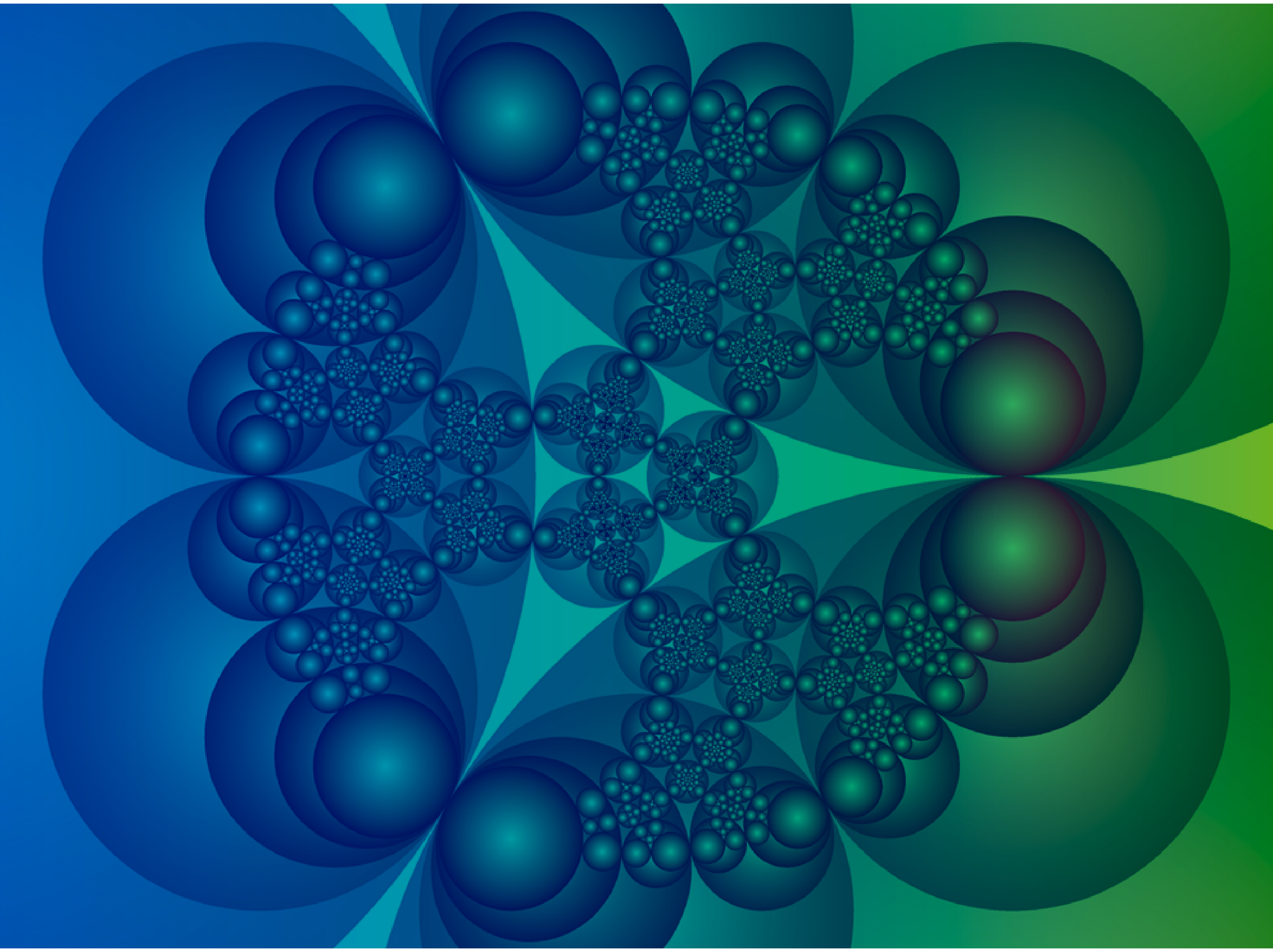
THE ERA OF MATHEMATICS

An Independent Review of Knowledge Exchange in the Mathematical Sciences

Professor Philip Bond



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Foreword

The UK is an increasingly innovative economy and the mathematical sciences play a key role both by providing solutions to numerous challenging problems and also by underpinning advances in other sciences, the social sciences and a wide range of key technologies.

Every day the mathematical sciences are used to solve otherwise intractable problems. On a daily basis we rely on the cryptography that secures our transactions over the internet and the optimal allocation of scarce resources, such as the radio spectrum which allows our mobile phones to work in crowded areas. The mathematical sciences underpin numerous scientific, technical and social advances that improve health and raise living standards. Genetic analysis relies on statistical methodologies, allowing improvements in human, animal and plant health. Machine learning, artificial intelligence (AI) and data science are dependent on mathematics to find patterns in complex datasets. The risk of pension and investment funds is managed and minimised using mathematical models and actuarial science. Operational research and statistical analysis underpin the high productivity of numerous industries and are extensively used by government and throughout the economy. Our national defence relies on advanced engineering and technology underpinned by sophisticated mathematical modelling, whilst our national security has long depended on sophisticated mathematics and remains a major employer of very high-calibre mathematicians. Flood management, meteorology, and the assessment of natural hazard risks rely on mathematical and statistical modelling. Mathematics is increasingly leveraged by those economies intending to compete by enabling digital, biomedical and environmental innovation to generate greater social and economic benefits.

The UK increasingly relies on innovation to provide a significant proportion of the productivity gains required to support rising standards of living. In consequence, the mathematical sciences are playing an ever-expanding role in generating innovation and impact and, via knowledge exchange, are adding substantial social and economic value to the UK.

I would like to thank everyone who has helped me in the course of the review. I have consulted widely both in academia and in industry and deeply appreciate the insights and help provided by the many contributors. I would particularly like to thank members of the Expert Review Committee and the Review Board for providing their time, insights and encouragement. I would also like to express my particular gratitude to Professor Dame Ottoline Leyser, Professor Sir Bernard Silverman, Professor Dame Julia Slingo, Professor Sir David Spiegelhalter and Professor Sir Alan Wilson for providing their expert, personal reflections on the use of the mathematical sciences in their fields. I would also like to thank the Knowledge Transfer Network Ltd (KTN) and the Engineering and Physical Sciences Research Council (EPSRC) for their logistical support and input into this review.

Professor Philip Bond

Preface

The Lord Stern of Brentford, Kt FRS FBA

We live in the era of mathematics. Its influence permeates economic and social activity and its influence and impact are profound. Yet its role is not well understood, we are not using it as well as we could and should, and we are investing too little. This report shows how we can do much better by reforming our institutions and investing more strongly, with potentially great benefits and relatively modest cost for the economy as a whole. There can be fewer more productive, creative and exciting investments than investing in mathematics.

This report sets out a very powerful and well-argued case for the extra investment and the new institutional initiatives that can accompany it. Taken together, they could be fundamental elements of the UK drive for productivity growth, innovation and a redefinition of its role in a world which is seeing very rapid changes in international and technological structures.

The report gives many examples. In my own discipline of economics I have seen the great power of mathematics but also the need for the new types of mathematical creativity. We have to do much better, for example, at understanding system risks, the dynamics of technology and increasing returns to scale, and the subtleties of human behaviour. Mathematics helps in all these, but it requires a deep collaboration with other disciplines. Examples abound too in climate change. Our modelling of global climates has depended greatly on mathematics, but again we must invest much more in the understanding of other risks outside the experience of *Homo sapiens*.

Cities are complex systems which we have undermined by poor management, particularly of space and motor vehicles. Again, technology is changing rapidly and we have great opportunities to manage much better and to create cities where we can move, breathe and be productive. The combination of mathematics with engineering and the social sciences will be fundamental.

New mathematical understanding does not come out of the ether. It requires investment in the pure mathematics that underlies all the rest, in the applications working with partners and other disciplines, in the people, particularly the young who will take it forward, and in understanding of mathematics from the top CEOs and ministers to those in the more technical areas who will do the 'hard graft.'

In 2016, at the request of the government I chaired a Review of the Research Excellence Framework used in UK universities. In our review report we argued for a perspective on impact which emphasises how a body of work can change understanding, change investment decisions, influence society and change policy. The 'Era of Mathematics' report is full of examples which meet this understanding of impact in a powerful way, from cybersecurity to social media to cities to food security.

This report makes a very powerful case for investing in mathematics. This is the era of mathematics and its influence will become still more intense. It is a discipline in which the UK can shine and lead. Now is the time to invest in its future in the UK.

Nicholas Stern

Executive summary and key recommendations

The mathematical sciences (MS) deliver significant social and economic impact in the UK. Mathematical tools and techniques lie at the heart of numerous industries, ranging from financial services to the special effects and computer-generated imagery (CGI) used in the film industry, underpin much of the technology used in national security and defence, and are now essential in the life sciences. Medical advances increasingly rely on mathematical data analytics, machine learning and process modelling, while medical imagers such as magnetic resonance imaging (MRI) scanners use algorithms directly derived from mathematical methods. Engineering and material sciences remain heavy users of mathematical methods, allowing the UK to remain a leader in numerous advanced engineering fields such as aerospace and Formula 1 motorsport.

Deloitte (2012) estimated that mathematics contributed over £200 billion annually to the UK economy in 2010, and that there were over 2.8 million individuals in employment directly due to mathematical science research in the UK, a figure that is still rising, with 6.9 million individuals in employment due to the wider ripple effects of mathematical science research in the UK. As increasingly mathematics-intensive industries, such as genomics and related medical sciences, grow in scale, so too will the contribution of mathematics to UK gross value added (GVA). Deloitte (2012) noted, 'The total GVA contribution in the UK in 2010 of mathematical science research is £556 billion, or over 40% of total GVA.'

In addition to the evidence from the Deloitte report on the economic impact of mathematical sciences research, data from recent EPSRC reports may be used to show that mathematical sciences research produces an outstanding rate of return on investment. The headline annual economic benefit for several disciplines principally within EPSRC's remit have been estimated in various publications to be: Engineering £280bn (EPSRC, 2015a, p.7), Physics £77bn (EPSRC/IOP/STFC 2014, p.3), Chemistry £258bn (EPSRC, 2015a, p.7), Mathematical Sciences £208bn (Deloitte, 2012). Furthermore the EPSRC report 'Investing in excellence, delivering impact for the UK: insights from the Research Excellence Framework 2014' (EPSRC, 2015b, p.13) noted that national spends on research in the period 2008-2013 were Engineering £3194m, Physics £2494m, Chemistry £1049m, Mathematical Sciences £354m. Although these numbers have been derived from a range of reports, a rate of return on investment as benefit-to-cost ratio may be estimated as follows: Engineering 88, Physics 31, Chemistry 246, and Mathematical Sciences 588.[1]

Our vision is for the UK to become a world leader in generating economic and social benefit from MS. As the UK redefines its place in the world by capitalising on its strengths, the application and impact of world-class UK mathematical sciences has a key role to play. Knowledge is generated globally, and a vibrant KE culture engaging internationally will remain critically important to the UK. The world requires 21st century mathematics to create 21st century technologies, and from smart cities to personalised medicine, new mathematics will lie at the heart of every major innovation.

British mathematical expertise has a long and distinguished history, as has the knowledge exchange that it has engendered. Sir Isaac Newton developed the calculus that underpins much modern applied science and engineering. Sir James Lighthill founded the field of aeroacoustics,

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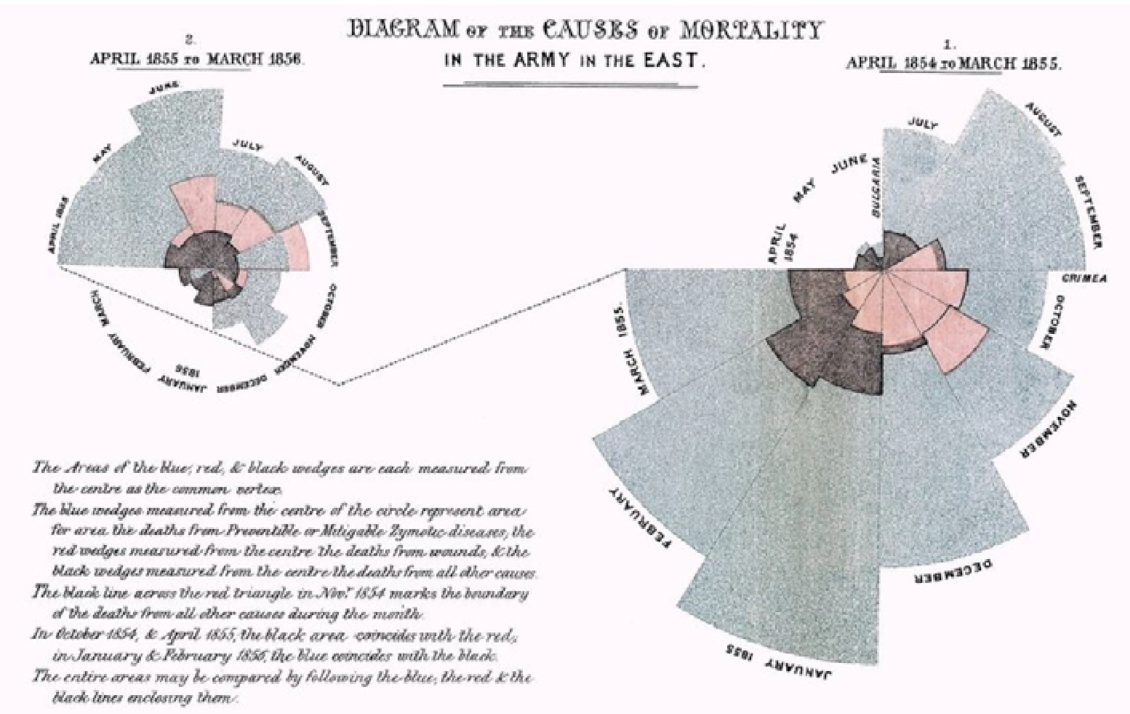
[1] *This analysis has been taken from a letter by Sir Adrian Smith to Professor Philip Nelson, EPSRC CEO, in 2016*

used to this day to reduce unwanted noise generated by aircraft in flight, and his initiatives in vertical take-off and supersonic flight directly led to the Harrier Jump Jet and Concorde. The mathematician Alan Turing developed the notion of a 'Turing machine' – or computer – to solve an outstanding problem of mathematical logic, the *Entscheidungsproblem* (decision problem). Turing's insights into mathematical computability took practical form in code-breaking work at Bletchley Park, where engineer Tommy Flowers, in conjunction with mathematician Bill Tutte, built the world's first electronic programmable computer, Colossus. In a previous generation mathematician Charles Babbage had also invented a programmable computer – the analytical engine. Like Turing and Tutte, he also contributed to national security by breaking the previously intractable Vigenere autokey cipher during the Crimean war. His collaborator, the mathematician Ada Lovelace, wrote the world's very first computer program in order to illustrate the function and potential of the analytical engine. The mathematician Florence Nightingale, the first woman elected as a member of the Statistical Society (now the Royal Statistical Society), revolutionised the practice of medicine by developing and applying statistical methodologies during the Crimean war. Her 'coxcomb' chart was the precursor of modern infographics (see Figure 1).

Figure 1: Florence Nightingale's polar 'coxcomb' diagram, developed during the Crimean war[2]



More recently Sir John Kingman Snr, a leading researcher in probability and genetics, developed the 'Kingman coalescence,' which models how gene variants may have originated from a common ancestor, and which now provides the basis of much genetic analysis, both human and in animals and plants. Simon Tavaré, recent head of the Cancer Research Institute in Cambridge, and colleagues have developed the sparse partitioning methodology now used throughout the world to analyse genes and progress our understanding of numerous forms of cancer. Data obtained by EPSRC and others attests to the world-class standard of current UK MS (see for example EPSRC 2017, p.12).

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[2] *Available at https://commons.wikimedia.org/wiki/File:Nightingale-mortality.jpg*

Factors that have contributed to the increasing importance of MS in recent years include:

- The power of MS in fields such as machine learning and artificial intelligence.
- Computational modelling, which is widely and increasingly used throughout the economy, as detailed in the recent Blackett review (Government Office for Science, 2018).
- MS as a key source of advanced algorithms.
- Optimisation as a source of productivity gains.
- The use of MS in the modelling of complex phenomena and complex systems, ranging from auxin transport in plants through to traffic flows, the weather and global warming.
- Increasing volumes of data and demand for reduced time-to-decision coupled with greater accuracy of decision-making under uncertainty.
- Statistical analysis supporting evidence-based decision-making in government and industry.
- Competitive pressure in industries such as automotive and aerospace manufacture and in financial markets.
- Regulatory pressures to improve safety and stability of systems including the banking system, nuclear plants and air-traffic control.
- The increasing need for rigorous analytic underpinnings in areas such as the social sciences, life sciences, quantum computing and data sciences.
- The requirement for accurate weather forecasting and flood prediction.
- The power of partial differential equations and their efficient numerical evaluation to model natural phenomena and industrial processes.
- Demand for strong cryptographic methods to secure data and communications.
- The need to model, understand and predict the behaviour of novel materials and meta-materials.
- Increasing sophistication of technologies required for national defence and national security.
- The need for adaptive cybersecurity defences.
- Insights into the structure and dynamics of data generated in domains ranging from modern medical systems to social media.
- The increased reliance on modelling in economics, agri-food, ecology, and across healthcare.
- The need to compress, transmit and secure internet data and transactions.
- The advent of large genomics databases.
- Increased demand for quantitative methods in areas such as policy, health and government.

This list is indicative and far from complete; demand for the mathematical sciences is increasing rapidly and requires both the development of novel mathematics and knowledge exchange in order to be wholly effective. New mathematics and knowledge exchange go hand-in-hand and this review, while focused on knowledge exchange, makes recommendations that incorporate the necessity for further infrastructure and an enhanced 'people pipeline' to achieve maximum impact by greatly enhancing the UK capacity to innovate throughout the mathematical sciences.

In summary, the outstanding power of mathematics to clarify complexity, to provide accurate real-time solutions and improve decision-making in complex and challenging contexts must be effectively harnessed. In an environment of significant pressure on government and industry to deliver superior services with constrained resources, there is ever-increasing necessity for all sectors of the UK economy to engage with the mathematical sciences.

To continue developing and delivering impactful mathematics, the UK will also need to further develop our skilled base of experts with a university education to engage with the service sector, industry and government. Mathematicians of the future will need additional skills to fully equip them to engage in an impactful way across a wide range of fields.

Skilled mathematicians of a high calibre are needed and they are in short supply. To support innovation in the UK a larger workforce both within and outside academia is required. New skills to enhance knowledge exchange will also be needed.

This review makes nine key recommendations.

KEY RECOMMENDATIONS

GOVERNANCE

I. An Academy for the Mathematical Sciences should be established in order to facilitate links between academia, government and industry. The Academy should act as the focal point and coordinating centre for the community and draw on the deep expertise of the existing learned societies.

SKILLS

II. Government and universities should create, at a minimum, 100 additional PhD places per year dedicated to training mathematical scientists looking to generate impact with their work. These PhDs should have a greater emphasis on breadth in training, with business and computer coding skills included in addition to deep mathematical expertise.

RESOURCES & INFRASTRUCTURE

III. To counter the underfunding of the MS research pipeline and adequately underpin MS in the UK, UK Research and Innovation (UKRI) should look to at least triple the funding going to MS across multiple Research Councils, including but not limited to EPSRC and Innovate UK.

IV. A national centre in impactful mathematics for the UK should be created to work with industry and government to drive mathematical research through to commercialisation. This could be based on existing models, such as the Fraunhofer Institute for Industrial Mathematics in Kaiserlautern or the UK Catapult network, suitably modified to provide national-level integration of low-TRL research from universities and to act as a national KE hub.

V. There should be at least one national centre, based on the Heilbronn Institute model, to better enable mathematicians focused on fundamental research to engage directly with government and/or industry.

REGIONAL

VI. Funds should be made available for regional KE centres and/or thematic KE networks following several successful models.

VII. Universities should have dedicated teams in mathematics departments to act as facilitators and KE translators. These should be connected to central KE functions within universities and coordinated through the National Academy.

GOVERNMENT

VIII. The Government Chief Scientific Advisor should, in collaboration with the Government Chief Statistician, review the access to, use of, and impact achieved by MS within government.

IX. The mathematical sciences should be encompassed in the HMRC definition of science and technology and included in the tax-credit scheme.

Review vision and full recommendations

> ‘He who does not employ mathematics for himself, will some day find it employed against himself.’
>
> — *Johann Friedrich Herbart*

The UK has an outstanding record in numerous sub-fields of the mathematical sciences (MS) including statistics, operational research, industrial mathematics, fluid mechanics, engineering mathematics, number theory, cryptography and many others. Generating impact requires both that this strong base is built upon and that interactions between MS and the wider economy are nurtured.

Increasing the impact of MS requires increasing awareness within government, the service sector and industry as to the potential power that mathematics has to deliver innovation. Mathematics is notable for being highly cost-effective. Our review has shown an uneven awareness and use of MS in UK industry and government, aligned to observations about the ‘long tail’ of underperforming UK companies. Increasing UK productivity by using operational research, statistics, optimisation, network science, industrial mathematics etc. will require increasing ease of access to MS and increasing awareness of MS within industry.

The importance of innovation in producing higher living standards is hard to overstate. The Chief Economist of the Bank of England, Andy Haldane, writes:

‘Since 1850 UK living standards, as measured by GDP per head, have risen roughly 20-fold, a huge gain. How much of that gain can be attributed to higher productivity? Well, if productivity had flat-lined over the period, UK living standards would only have doubled. Or, put differently, in the absence of productivity growth, UK living standards would be an order of magnitude lower today, stuck at late-Victorian levels. A more refined way of reaching the same conclusion is to decompose growth into the contribution from inputs into the production process – labour and capital – and the contribution from improvements in the efficiency with which these inputs are used – so called Total Factor Productivity (TFP). This suggests movements in TFP have accounted for the lion’s share of both the growth and variation in living standards since at least the mid-18th century.’

Approximately 50% of the growth in productivity in the UK in recent decades has derived from the development and uptake of innovative processes and technologies (OECD, 2015). In other words, innovation across the entire economy is a fundamental driver of living standards for the UK and mathematics is arguably the single most pervasive and powerful of all drivers of innovation in the world today.

Many of the world’s largest and most innovative organisations rely on mathematical advances to make them world-leading, for example:

- Google was founded on a mathematical algorithm known as the Page Rank Algorithm, which estimates the leading eigenvalue of a huge matrix representation of all links in the World Wide Web.
- Financial asset managers rely on mathematical models for the pricing of options, optimisation of asset portfolios and for risk management.
- Airlines schedule flights using methods pioneered in operational research.
- Walmart famously uses predictive algorithms to dominate US retail.

Driving impact from academia into the economy lies at the heart of the UK government's Industrial Strategy. We believe that taking action now will lead to substantial gains in impact and innovation generated by MS throughout the economy. This review makes 26 recommendations, listed in full below, that describe how government, industry and academia can engage to ensure that the UK Industrial Strategy is supported now and in the future by this powerful engine of innovation.

FULL LIST OF RECOMMENDATIONS

GOVERNANCE – recommendations for the entire mathematical sciences community

1. An Academy for the Mathematical Sciences should be established in order to facilitate links between academia, government and industry. The Academy should act as the focal point and coordinating centre for the community and draw on the deep expertise of the existing learned societies.
2. The means to structure, streamline and raise awareness of the existing KE support mechanisms that are available should be generated.
3. Existing mechanisms for knowledge exchange (KE) initiation should be made more robust and expanded in scope and capacity. Mechanisms should be put in place that make it straightforward for both industry and academics to find appropriate expertise.
4. Awareness should be raised within the mathematical sciences community of wider research challenges and societal challenges (including the sustainable development goals addressed by the Global Challenges Research Fund, GCRF) and deeper integration of mathematics should be promoted within industrial challenges (including the Industrial Strategy Challenge Fund, ISCF).
5. A more systematic and coordinated approach needs to be adopted to make new and maintain existing KE contacts and to track the outcomes and impacts of KE activities.

SKILLS – recommendations for the higher education sector

6. Government and universities should create, at a minimum, 100 additional PhD places per year dedicated to training mathematical scientists looking to generate impact with their work. These PhDs should have a greater emphasis on breadth in training, with business and computer coding skills included in addition to deep mathematical expertise.

7. Better provision should be made for early-stage training of mathematicians in KE and problem framing/solving, at undergraduate, masters, PhD and postdoctoral levels. Project/thesis work should increasingly be undertaken in partnership with government or a commercial organisation, with increasing use of internship as a mechanism.
8. All mathematics students should acquire a working knowledge of at least one programming language.
9. Incentives should be created to enable two-way movement of researchers between academia, industry and government.
10. Mechanisms to generate systematic and long-term relationship building and engagement with alumni should be created.
11. MS masters and PhD students with an interest in specialist research areas should have the opportunity to engage with national-level initiatives, e.g. researchers in artificial intelligence (AI) with the Alan Turing Institute, in operational research (OR) with government and Dstl, in materials modelling with the Royce Institute, and various subjects with specific Catapult centres.
12. KE activities should be fully integrated into MS academic careers and career progression. This should include consideration of KE in academic appointment and promotion criteria, as well as mechanisms to incentivize and support KE activities. Mechanisms should include KE accolades and buy-out of teaching time for academics who complete an industry placement to ensure that academic research productivity is maintained.

NATIONAL RESOURCES & INFRASTRUCTURE – recommendations for new funding and infrastructure

13. To counter the underfunding of the MS research pipeline and adequately underpin MS in the UK, UK Research and Innovation (UKRI) should look to at least triple the funding going to MS across multiple Research Councils, including but not limited to EPSRC and Innovate UK.
14. A national centre in impactful mathematics for the UK should be created to work with industry and government to drive mathematical research through to commercialisation. This could be based on existing models, such as the Fraunhofer Institute for Industrial Mathematics in Kaiserlautern or the UK Catapult network, suitably modified to provide national-level integration of low-TRL research from universities and to act as a national KE hub.
15. There should be at least one national centre, based on the Heilbronn Institute model, to better enable mathematicians focused on fundamental research to engage directly with government and/or industry.
16. Resources for workshops with industry should be broadened and increased. In particular the Mathematical Study Groups with Industry should be expanded in scope.
17. Strong incentives should be put in place for cross-disciplinary work between the mathematical sciences and other disciplines.

REGIONAL SUPPORT – recommendations seeking to boost local economies

18. Funds should be made available for regional KE centres and/or thematic KE networks following successful models such as the Turing Gateway to Mathematics, the UK fluids network (ERCOFTAC), the University of Bath's IMI and the University of Oxford's OCIAM.

19. Universities should have dedicated teams in mathematics departments to act as facilitators and KE translators. These should be connected to central KE functions within universities and coordinated through the National Academy.

20. Innovate UK should actively seek to create mechanisms within ISCF and the Small Business Research Initiative (SBRI) that encourage industry to engage and form partnerships with MS experts.

21. Incentives for academic engagement with local SMEs should be created.

22. PhD training centres and other centres of excellence should integrate knowledge exchange more tightly and seek to interact more extensively with local SMEs and larger businesses.

GOVERNMENT -recommendations for central government function

23. The Government Chief Scientific Advisor should, in collaboration with the Government Chief Statistician, review the access to, use of, and impact achieved by MS within government.

24. The mathematical sciences should be encompassed in the HMRC definition of science and technology and included in the tax-credit scheme.

25. Deeper links between key government users of MS and academic departments should be encouraged.

26. Government should actively engage with MS to examine means to utilise MS to improve productivity across the economy.

SECTION 1:

Mathematical sciences, knowledge exchange and objectives of the review

> ‘The Universe … cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language.’
>
> — *Galileo Galilei, The Assayer (Il Saggiatore), 1623*

1.1 What are the Mathematical Sciences?

Mathematics is a discipline that enables us to understand and generate patterns and structures and develops powerful tools for working with them. Mathematicians actively engage in creating new tools and new mathematical structures and utilise a combination of new and existing tools to solve problems or further elucidate understanding. It is essential not only for understanding physics but increasingly for economics, geography, plant biology, climatology, artificial intelligence and numerous other fields which are being ‘written in mathematical language.’

The patterns developed in mathematics are used to solve numerous hard real-world problems. For example, elliptic curves are used to cryptographically secure internet traffic and mobile phone calls. Patterns found in the world around us can also be understood, clarified and used to predict real-world phenomena. For example, the behaviour of the antenna on our mobile phones is predicted and optimised using mathematics as are weather forecasts or flood predictions. The future of personalised medicine depends upon insights gained by mathematical analysis of vast genomics databases. The creation of new patterns and structures allows deeper and more rigorous insight into fields such as data science, AI and genetic analysis. The tools created by mathematicians can be converted to practical use, often in the form of models or algorithms, which are embedded within just about every modern technology and socio-technical system.

Historically, mathematicians have engaged in solving both theoretical or ‘fundamental’ and practical problems. Over time, as specialisation has occurred, there has been a tendency to separate some branches of mathematics. Nevertheless, virtually all areas of mathematics overlap at various levels and in this review we include fields ranging from geometry and topology through statistics and operational research to industrial mathematics. All have an important role to play. We have used the term ‘industry’ to include not only manufacturing but also wider commercial activity. Indeed, the service sector, and particularly financial markets, remain significant users of mathematical methods drawn from virtually every discipline.

1.2 Impactful Mathematics

Mathematics is often considered as split into ‘pure’ or ‘fundamental’ and ‘applied’ mathematics. These historical distinctions are not necessarily helpful for the purposes of this review. Our concern is primarily with *impactful mathematics*. This is mathematics that has real-world impact. Both ‘pure/fundamental’ and ‘applied’ mathematics can be, and are, impactful. We refer to ‘impactful mathematics’ as any mathematical method that has practical application and generates societal and/or economic value.

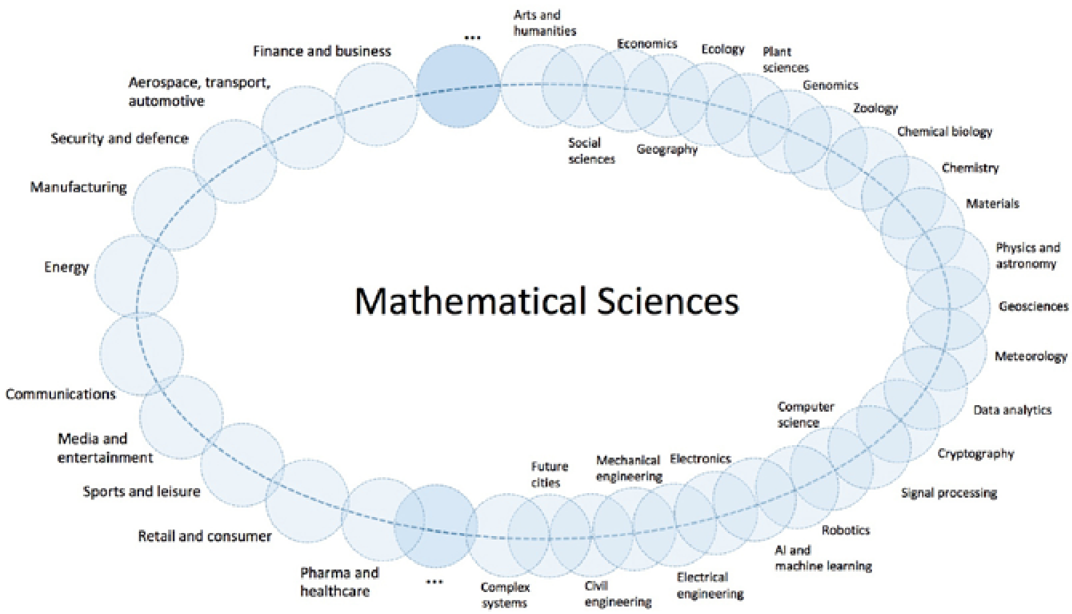
By way of example, graph theory has often been labelled as 'pure' within the mathematics community, but has found impact in a wide range of fields. Graph databases, often used by social media websites, represent one well-known example. Number theory is also often labelled as 'pure' but is essential in the development of modern cryptographic systems; similarly 'pure' harmonic analysis underlies much modern signal processing and image analysis.

Examples of impactful MS in our everyday lives are ubiquitous but often hidden from view. Our journey to work is optimised by route-planning algorithms using large graph databases to plan the route efficiently. Aircraft noise overhead is reduced thanks to better modelling of turbulent airflows, while the aircraft are safely tracked using sophisticated mathematical algorithms. Our mobile phones use cryptographic algorithms to secure data, compress images and movies and even to make our selfies look better. The submarines that protect our shores run 'silent and deep' due to advanced engineering mathematics. Sophisticated mathematics underpins our national security and underlies many advances in cybersecurity which help to protect businesses and institutions and to keep our private data secure. Mathematical modelling is used to design 'smart cities,' to increase car-engine efficiency and optimise hybrid engines, and allows wind turbines to operate at peak efficiency. Our Olympic athletes perform using equipment that has been modelled and optimised mathematically, helping GB medal-winning performances in sports ranging from track cycling to skeleton bob. Much mathematics is embedded in the form of algorithms, or structural designs, that are hidden within engineered systems. As a result the mathematical content of the world around us is sometimes less than obvious to a user, but essential nonetheless.

Impactful mathematics appears throughout the economy. Companies use statistics to plan and to manage risk, operational research is used to streamline operations, reducing costs and improving productivity. Government statistics underpin effective evidence-based decision-making. Financial services, security, defence, health, manufacturing, transport, film-making, and many other sectors all make use of many fields within the mathematical sciences. Developments in genomics, data science, economics, physics, quantum computing, biology, advanced engineering, epidemiology, zoology, sociology, geography, ecology, climate science, cybersecurity, social media analytics and numerous other fields all require the use not only of existing mathematical methods, but also the development of new, more powerful mathematical tools to continually spur advances and innovation. The breadth of impact of MS may be seen in EPSRC's report on insights from the 2014 Research Excellence Framework evaluation (EPSRC, 2015b), which found linkages to all 22 industry sectors. Some of the linkages and the relationship of MS with other disciplines are shown in Figure 2.

> 'There are huge opportunities for research into the best ways of using data in innovative tools of direct benefit to people's lives. A simple example concerns calculators for providing prognostic assessments for recurrence and mortality from cancers. Such applications take advantage of the massive datasets that have been accumulated by UK Cancer Registries.'
>
> – *Professor Sir David Spiegelhalter*

Figure 2. Some linkages of the mathematical sciences to other academic fields and industry sectors[3]



1.3 The Power of the Mathematical Sciences

A number of personal reflections from invited contributors are given throughout this review to illustrate the power and diverse applicability of the mathematical sciences to some of the most important challenges facing humanity today. These contributions cover a range of domains including food security and plant sciences, weather and climate prediction, future cities, and government use of the mathematical sciences.

1.4 What is Knowledge Exchange in the Mathematical Sciences?

This report disputes passive visions of mathematics and impact, such as that articulated by the great French mathematician, Jean-Pierre Serre, who wrote, '*As for the place of mathematics in relation to other sciences, mathematics can be seen as a big warehouse full of shelves. Mathematicians put things on the shelves and guarantee that they are true. They also explain how to use them and how to reconstruct them. Other sciences come and help themselves from the shelves; mathematicians are not concerned with what they do or with what they have taken. This metaphor is rather coarse, but it reflects the situation well enough.*'

In contrast, we argue that knowledge exchange in the mathematical sciences makes mathematical knowledge far more impactful than the passive shelf-stacking model referred to by Serre. Knowledge exchange seeks to actively engage with other research fields, industry and government. It aims to understand and be concerned with the end use of mathematics. It seeks to understand potential uses of mathematics so that superior tools, techniques and algorithms may be developed for them. This often requires the creation of new mathematics. Through translation KE takes 'the books off the shelf' and uses them to solve real-world problems. Real-world applications have always provided challenges which can only be solved by creating new or sharper mathematical tools. These new tools are then available for use on other problems. In brief, the goal of KE is to maximise the impact of the mathematical sciences.

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[3]*Adapted from National Research Council (2013), Figure 3.2*

The UK has a number of excellent centres engaging in knowledge exchange. We cite, by way of example, two internationally renowned UK-based centres here.

The **Isaac Newton Institute for Mathematical Sciences (INI)**, based in Cambridge, is a leading international research institute serving the whole UK community. It hosts events across all areas of the mathematical sciences as well as interdisciplinary topics, including long-term scientific programmes, workshops, satellite meetings, 'open for business' events, scoping meetings and follow-up activities. Its impact-acceleration arm is the Turing Gateway to Mathematics (TGM), which acts as a vehicle for knowledge exchange between the mathematical sciences and potential users of mathematics from industry, commerce, government including regulators and policy makers, business and other academic disciplines. INI and TGM collectively host over 2500 visitors per annum.

The **Heilbronn Institute for Mathematical Research** allows mathematicians working on fundamental research, more particularly number theory, to spend half of their time focussing on challenges for national security and half of their time on academic research. Heilbronn research fellowships are highly sought-after and extremely productive. They are an excellent example of 'fundamental' mathematicians *engaging* with government directly, *understanding* the applications and creating *new mathematics* in order to tackle hard problems that directly impact and benefit our lives by enhancing national security. The Heilbronn Institute model is discussed further in Section Two.

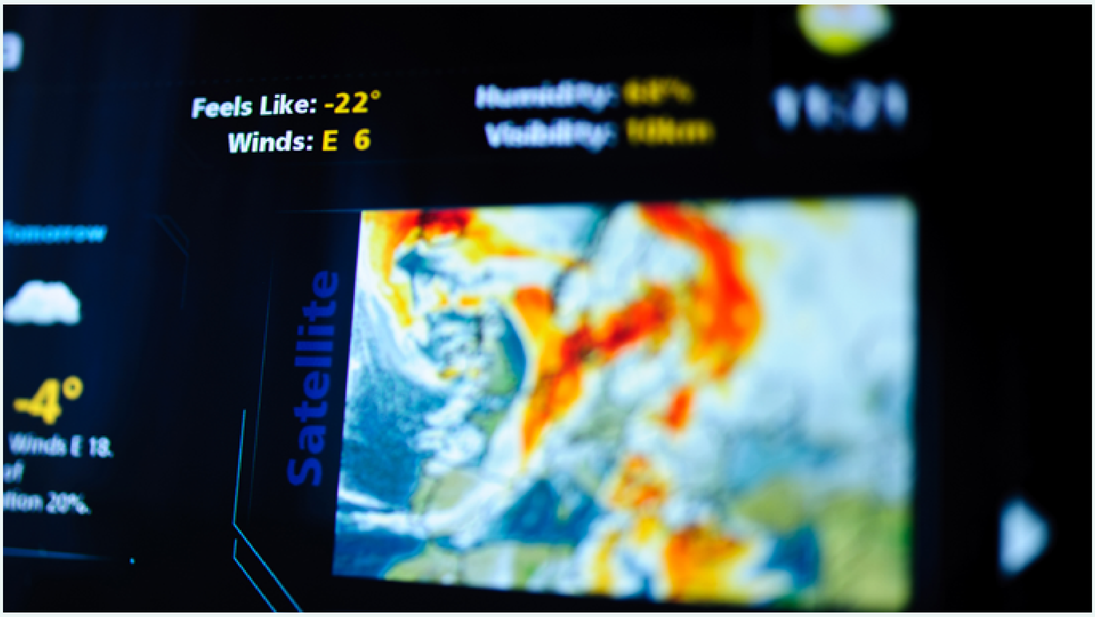
There is an important distinction to be made regarding the underlying funding models for the Heilbronn Institute and the Isaac Newton Institute. Both are national centres, but Heilbronn is funded by GCHQ, directs much of its effort towards GCHQ's agenda, and has a subset of the mathematical sciences as its remit. INI is largely funded by EPSRC/UKRI and is a national research centre for the entire discipline and its applications.

These and other excellent UK institutions serve as valuable models of effective and impactful KE. Increasing the scale, scope and geographic reach of such institutions in the UK will be required if the full potential of UK mathematical impact is to be realised, and several recommendations in this review are aimed at enhancing and augmenting the existing institutional infrastructure to achieve this goal.

A key concept in modern KE is that it is a two- or multiple-way exchange of knowledge between mathematical scientists in academia and others, including but not limited to industry, service providers and policymakers. This definition encompasses the following categories (adapted from Lawson *et al*., 2016):

- **Working with industry**, including networks, joint projects, secondments, consultancy;
- **Dissemination** of findings;
- **Training** – staff and students, curriculum development;
- **Public engagement**;
- **Working with other disciplines**, including networks, joint projects, joint publications, secondments;
- **Commercialisation**, including patents, licences, spin-out companies.
- **Policy influence**, for example membership of advisory forums.

Mathematics in Weather and Climate Prediction - Professor Dame Julia Slingo, formerly Chief Scientist, Met Office



Weather forecasts are part of our daily lives and, at times of severe weather, they are vital for keeping us safe, helping us to prepare for the impacts on our lives, and enabling critical services, such as transport and energy providers, to be ready for action. At the same time, we have become acutely aware that our climate is changing and that this is largely due to our actions. We are able to look into the future and make predictions of what our climate might be like in the decades ahead, depending on what actions we choose to take. Our ability to forecast the weather days in advance, and to predict our future climate and how it will change due to human activities, owes much to mathematics.

Weather forecasting began over 150 years ago with empirical models based on observations, but with the advent of computers in the 1950s the possibility of numerical weather prediction, based on simulating the weather from first principles using fundamental physical laws, such as Newton's Laws of Motion, radiative transfer theory and moist thermodynamics, began to be explored. Today numerical weather prediction represents one of the most complex applications of supercomputing, and the codes that produce our weather and climate predictions typically run to over a million lines of code. Over the years, the same codes have been adapted and extended to simulate the climate system (for example, by adding a fully interactive ocean) and, increasingly, the Earth system (for example, by adding biogeochemical cycles such as carbon).

So where does mathematics come in? The circulation of the atmosphere and oceans is described by a system of partial differential equations (PDEs) on a sphere. These cannot be solved analytically and have to be discretized on a grid and solved numerically. The methods for doing this involve complex mathematics to ensure that solutions are stable, efficient and accurate, and that the basic properties of the fluid (such as mass and energy) are conserved. Importantly the solver has to be computationally efficient, so that the forecast can be produced within the time constraints of the forecast's shelf-life. Here the length of the time step in the forward model is critical, with semi-implicit methods being developed to remove fast, high-frequency waves, such as sound and gravity waves, which have little meteorological importance. The choice of grid on which to integrate the equations, from the traditional latitude-longitude grid to more exotic unstructured grids, also has a big impact on the accuracy and efficiency of the numerical method.

Beyond the numerical core of weather and climate models, mathematics is also fundamental to how we initialize the predictions and represent uncertainty in the forecasts. Forecasts start from the observed state of the atmosphere today, and then the model integrates this initial condition forward in time. Observations come from a myriad of platforms – satellites, aircraft, ships, sondes, etc. – which are highly heterogeneous in space and time. To produce a three-dimensional, fully consistent analysis of the current state of the atmosphere involves the process of data assimilation, in which the observations are incorporated into the forecast (background) model in an optimized way which preserves the balanced state of the physical system and seeks to minimize the errors inherent in the observations and the background model. Mathematics is at the core of data assimilation and advances in forecast skill are closely tied to advances in data assimilation, which have enabled the optimal use of observations within the framework of the numerical model.

Uncertainty is an inherent property of the fluid motions of the atmosphere and oceans. This was recognized in 1963 by Ed Lorenz in his seminal paper on 'Deterministic non-periodic flow' in which he introduced the concept of the atmosphere as a chaotic system subject to small perturbations that grow through non-linear processes to influence the larger scale – 'the flap of a seagull's wings may forever change the course of the weather.' The concept of the weather and climate as chaotic systems has had a profound impact on the way in which forecasting has evolved over recent decades. No longer do we produce a single, deterministic forecast, but instead we perform an ensemble of forecasts that seek to capture the plausible range of future states. This enables the probability of certain outcomes to be assessed so that the user can make a value judgement on how best to minimize the risks. The design of the ensemble, the methods used to optimize the spread of the ensemble, and the assessment of the reliability and skill of the system, all require mathematics.

And finally, it is worth noting the big-data challenge of weather and climate prediction. Each day the Met Office produces over 10 terabytes of forecast data which have to be analysed, interpreted and verified. Mathematics again plays a critical role and will increasingly do so as data volumes increase. The potential to use AI and machine learning techniques to extract and add value to forecast information is one new avenue for mathematics in the future.

Over the decades the Met Office has been at the forefront of international developments in weather and climate prediction. This is in part because it has a world-class research department that draws heavily on mathematical skills and employs high quality mathematicians. Increasingly it engages with the UK academic community and the Research Councils to draw the best mathematicians into its science and technology challenges, such as developing the next-generation weather and climate-prediction systems that can exploit emerging supercomputing architectures.

1.5 Objectives and Structure of this Review

This review followed from a research community meeting on KE in the mathematical sciences convened at the International Centre for Mathematical Sciences (ICMS) in autumn 2015. An Expert Review Committee was established to gather and analyse evidence following an open Expressions of Interest process in autumn 2016 and began work in February 2017. The Committee members are listed in Annex 2 to this report. A Review Board of senior individuals from the mathematical sciences research community, industry and the public sector with appropriate experience and international standing, chaired by Professor Philip Bond, was constituted to provide strategic advice to the review and to help communicate its findings to relevant audiences. The Board membership and its Terms of Reference are given in Annex 3. The Engineering and Physical Sciences Research Council (EPSRC) and the Knowledge Transfer Network (KTN) provided advice and operational support to the review.

The review set out to address the following key questions:

- What does successful and unsuccessful KE in the mathematical sciences look like? What are the characteristics of success for all stakeholders?
- What are the mechanisms and enablers to support KE in the mathematical sciences?
- What are the career incentives for researchers to engage in KE-related activities in the mathematical sciences?
- How can mathematical scientists connect with research users and maintain genuine relationships?
- How can the mathematical sciences 'brand' be enhanced and associated with solutions amongst end users of research?
- What are the opportunities and risks for KE in the mathematical sciences?

Emerging from these questions the review has also considered whether the mathematical sciences have access to sufficient resources to deliver the key underpinning role described above, and what actions may be taken to strengthen UK support for and underpin best practice in KE. The Review Committee and Board considered a broad range of evidence to inform their discussions; a brief overview is provided below. Additional details of the evidence gathered for the review are given in Annexes 5 and 6 to this report, which are published on the EPSRC website.[4]

The strands of evidence gathered to address the questions above may be summarised as follows.

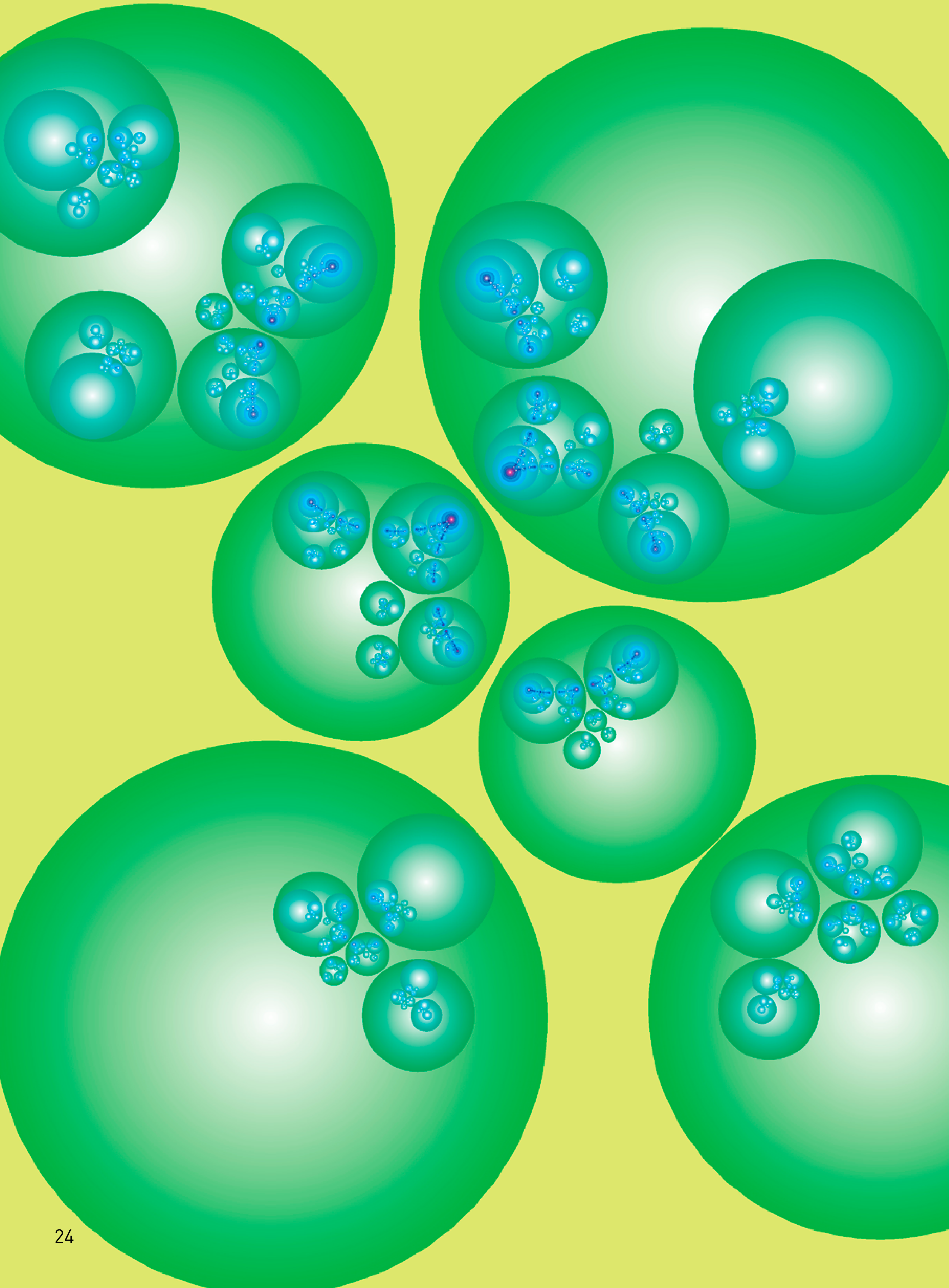
1. A call for evidence inviting institution-level views on the mathematical sciences knowledge exchange landscape in the UK (25 responses, listed in Annex 4).
2. A survey seeking respondents' views on mathematical sciences knowledge exchange enablers, expectations and experiences (see Section Three). We received 351 responses: 143 from industry stakeholders, 170 from academia and 38 from the public sector. Further details are given in Annex 5.
3. A survey seeking case studies of knowledge exchange activities (149 responses, listed in Annex 5).

[4] *https://epsrc.ukri.org/newsevents/pubs/era-of-maths*
https://epsrc.ukri.org/newsevents/pubs/era-of-maths-annex5
https://epsrc.ukri.org/newsevents/pubs/era-of-maths-annex6

4. Two community-engagement workshops to consider the potential key messages of the report, taking into account the evidence gathered through the other work strands. Full details and a list of workshop attendees may be found in Annex 6.

5. A sub-group of the review committee also gathered evidence to inform international comparisons with the UK's mathematical sciences knowledge exchange landscape. The results of this work strand are presented in Section Two.

Unattributed quotations in Sections Two and Three of this report are extracts from the responses received to the review call for evidence.



SECTION 2:

UK landscape for knowledge exchange in the mathematical sciences

2.1 Ways and means

Knowledge exchange provides powerful impact for the UK economy when underpinned by strong supportive mechanisms and appropriate institutions and institutional incentives. The current landscape is complex, involving more than forty different routes for promoting and supporting knowledge exchange in the mathematical sciences.

These mechanisms include a mixture of institutional-level support, external mechanisms and national infrastructure. At the institutional level support mechanisms include the Higher Education Innovation Fund (HEIF) and EPSRC's Impact Acceleration Accounts. External routes include, for example, studentships and Study Groups. At the national level dedicated entities/ institutes include the INI, Heilbronn Institute for Mathematical Research and the International Centre for Mathematical Sciences (ICMS; see also EPSRC, 2015c).

> ‘One-size-fits-all policies for technology transfer do not work; universities, technologies and places vary […] Focussing on spin-outs as the measure of success in knowledge exchange (KE) in universities gives a distorted picture, as universities need to pursue the most appropriate route to impact for the particular research/technology.’
>
> – *(HEFCE, 2016)*

> ‘Knowledge exchange is delivered in a myriad of different ways supported through universities, funding bodies, learned societies, academic and industrial networks and more.’

The five specific mechanisms for KE support most commonly cited in our review call for evidence responses are listed in Table 1. Dissemination routes, such as conference presentations, and public engagement of research are of course also important routes for KE and impact. Table 1 does not take into account the personal efforts of individuals in KE; this point is discussed in Section Three.

Table 1. The five most commonly cited KE support mechanisms from the review call for evidence

KE support route	Description	Reference links to further information
Industrial CASE (ICASE) PhD studentships	Industrial Cooperative Awards in Science & Technology (CASE) provide funding for PhD studentships where businesses take the lead in arranging projects with an academic partner of their choice.	https://www.epsrc.ac.uk/ skills/ students/coll/icase/intro/ http://www.smithinst.co.uk/
Knowledge Transfer Network (KTN) Ltd activities	The Knowledge Transfer Network is funded by Innovate UK as their network partner and also provides innovation networking for other funders in line with its mission to drive UK growth.	https://www.ktn-uk.co.uk/
Mathematics Study Groups	Initiated in Oxford in 1968, Study Groups with Industry provide a forum for industrial scientists to work alongside academic mathematicians on problems of direct industrial relevance.	http://www.maths-in-industry.org/
Turing Gateway to Mathematics activities	The Turing Gateway to Mathematics (TGM) is an impact acceleration initiative of the Isaac Newton Institute (INI) based at the University of Cambridge.	https://www.turing-gateway.cam.ac.uk/ https://www.newton.ac.uk/
Knowledge Transfer Partnerships	The Knowledge Transfer Partnership (KTP) scheme aims to help businesses in the UK to innovate and grow by linking them with an academic or research organisation and a graduate.	https://www.gov.uk/guidance/ knowledge-transfer-partnerships-what-they-are-and-how-to-apply https://info.ktponline.org.uk/ action/ search/current.aspx

However, we have found in this review that it is commonly perceived in the research community that there are few clear routes open for the exploitation of mathematical sciences research not funded by EPSRC, i.e. not eligible for follow-on support through EPSRC funding routes. Furthermore, the scale of funding available for mathematical sciences KE in practice often follows the scale of the research funding, and is therefore in effect disproportionately small compared to other STEM disciplines.[5]

The diversity of current routes in support of KE – both academia- and industry-led – is also potentially difficult to navigate, making it difficult for individual researchers and companies to know where to start. The Warry report (Research Councils UK, 2006) and more recently the Dowling review across all disciplines (Dowling, 2015) have noted similar difficulties in other fields and have recommended that procedures be simplified and clarified. As the mathematical sciences are highly diverse, the need for clarity becomes pressing, and a number of the recommendations in this review are designed to address this issue.

[5]In 2016-17 EPSRC spent £19.7 million on research through its Mathematical Sciences Theme, compared with, for example, £70.7 million in Engineering and £82.9 million in Physical Sciences (EPSRC, 2017)

> The breadth and variety of mechanisms ..., and the fact that they are tailored to meet particular research areas (whether it be early stage or more applied), is a real strength of the support for knowledge exchange in the UK, particularly in the mathematical sciences.

> There are numerous bodies and support mechanisms out there, and it can be difficult/time-consuming to work out which are the best ones to utilise for a particular project/idea.

We also note that positive views of particular support routes do not necessarily equate to high take-up of these routes by the mathematical sciences research community. For instance, analysis of EPSRC ICASE data for PhD students starting in 2016 showed that a low proportion of the projects were led by mathematical sciences departments compared with, for example, engineering departments. Recognising the limitations of a simple analysis based on department name, this proportion was significantly smaller (by a factor of two) than the relative EPSRC research spend on the two discipline groups (see footnote 5). This mechanism is industry-led, so it is important to note that the placement of students depends on industry awareness of and relationships with academic partners. However, raising academic awareness of support routes is essential, such as funding for KE workshops and meeting places for academia and industry, which are available from Innovate UK and the Knowledge Transfer Network, for example. A national-level infrastructure for supporting KE in the mathematical sciences is required.

Recommendation: The means to structure, streamline and raise awareness of the existing KE support mechanisms that are available should be generated.

Knowledge exchange in the mathematical sciences is broader than industry engagement and commercialisation, including, for example, routes to impact via engagement with other academic disciplines (see also Meagher and Martin, 2017). The Nurse review (Nurse, 2015) that led to the creation of UKRI addressed the increasing importance of interdisciplinary work in the UK. The mathematical sciences are arguably the broadest and most effective cross-cutting and interdisciplinary of all subjects. If the full potential of UK mathematical sciences in empowering cross-cutting research is to be realised new mechanisms for engagement within universities and across the academic-industry landscape are required. This point is considered further in Section Three of this report.

2.2 Incentives and Enablers in the UK Landscape

Funding support for different scales and timescales of activity (see also Section Three) is important for fostering successful KE, for example resources for networking, scoping and pilot projects. At a higher level the Research Councils' and Higher Education Funding Councils' increased emphasis on research impacts over the last ten years has been a positive influence in encouraging mathematical sciences researchers to consider the wider benefits and implications of their research. There are a number of enablers and incentives available to higher education institutions (HEIs) for researchers to engage in KE; these are reviewed in detail by Dowling (2015). A subset of these enablers is discussed in brief below, along with their relationship to MS. More detail on incentives for stakeholders outside HEIs is given in Section 3.2.

A major incentive for UK HEIs to engage in KE is via the 20% impact component of the Research Excellence Framework (REF). Most quality-related research (QR) funding is currently allocated on the basis of the REF. The 2014 REF required universities to submit responses which were assessed against the following criteria: the quality of research output (65% weighting); the impact of research beyond academia (20%); and the research environment (15%). Initial decisions for the next REF in 2021 include raising the impact weighting to 25% (REF, 2017). In assessing impact, the 2014 REF Mathematical Science sub-panel observed that *'about 50% of case studies could be characterised as having been underpinned, mainly though not exclusively, by research in statistics and operational research, 45% by applied mathematics, and 5% by pure mathematics. However, many different aspects of the mathematical sciences were seen to contribute to a given impact case study,'* (REF, 2015).

In England, Research England (previously HEFCE) provides funds via Higher Education Innovation Funding (HEIF) to HEIs to engage in KE, based largely on the annual Higher Education Business and Community Interaction (HEBCI) survey.[6] The HEBCI records activities including collaborative research, consultancy, facilities and equipment services, continuing professional development, development programmes, and income from intellectual property. As discussed in Meagher and Martin (2017), these indicators have not historically worked well for MS, so that HEIs are not incentivised to invest HEIF money into MS departments. However, with a further £40 million provided by government through HEIF to support delivery of the Industrial Strategy in 2017-18, and likely changes to the method used to distribute this funding following the Knowledge Exchange Framework (KEF) consultation, we must ensure that this mechanism works effectively for MS.

Clearly there are also incentives via UKRI for HEIs to engage in KE; all UKRI research grants require that careful consideration be given by researchers from the conception of a project to how the impacts of the research will be maximised.[7] Impact Acceleration Accounts (IAAs) are available from several Research Councils to support KE and innovation activities. IAAs are allocated as a block grant to individual research organisations across the UK, based on their previous success in securing competitively won research funding from the relevant Council. IAAs are 'particularly valued for the speed with which the funding can be mobilised and deployed' (Dowling, 2015).

[6] *See for example http://www.hefce.ac.uk/media/HEFCE,2014/Content/Pubs/2017/201723/HEFCE2017_23.pdf*
[7] *See for example https://www.epsrc.ac.uk/funding/applicationprocess/preparing/impactguidance/*

Recommendation: Existing mechanisms for KE initiation should be made more robust and expanded in scope and capacity. Mechanisms should be put in place that make it straightforward for both industry and academics to find appropriate expertise.

2.3 People Pipeline – Careers and KE in the UK

Demand for mathematical expertise across a wide range of subjects is booming: in addition to perennial demand for first-rate mathematical talent from financial markets, developing fields such as AI and machine learning, genomics, autonomous vehicle development, robotics, data science, the digital economy and many others are creating highly paid jobs for appropriately skilled people. This in turn places additional burden on mathematics departments; see, for example, CMS (2016). To address this issue we recommend creating a cohort of PhDs who receive not only an excellent mathematical education, but also have the opportunity to learn business skills and engage with industry at an earlier stage. Furthermore we recommend breaking down traditional barriers within mathematics and having Centres for Doctoral Training (CDTs) which mix 'foundation/pure,' 'applied' mathematics and statistics. Mathematics from 'both sides of the divide' can and should be used to generate impactful mathematics and solve hard problems of social and economic value. Historically the very best mathematicians did both fundamental mathematics and deliberately impactful mathematics. We need a new generation of intellectually flexible mathematicians trained to engage with 21st century challenges both through fundamental and through deliberately impactful mathematics. This point was also made in the 2010 International Review of the Mathematical Sciences (IRMS; see EPSRC, 2011):

> 'UK PhD students are too narrowly educated and this issue is likely to become more pressing as globalisation increases. [...] Deep knowledge of a research area in the mathematical sciences is essential but, since the problems faced by industry are constantly changing, flexibility and adaptability are equally important in the long run.'

The CMS people pipeline report (CMS, 2015) estimated the routes and destinations of applicants for higher degrees in the mathematical sciences. This was graphically mapped in the report, reproduced here in Figure 3. The CMS analysis showed that in 2013 the UK produced 655 MS graduates, 84% of whom were in full-time employment six months after graduation. In 2013 the number of people in jobs where a mathematical science qualification was essential was estimated to be two million – 7% of the total UK workforce.

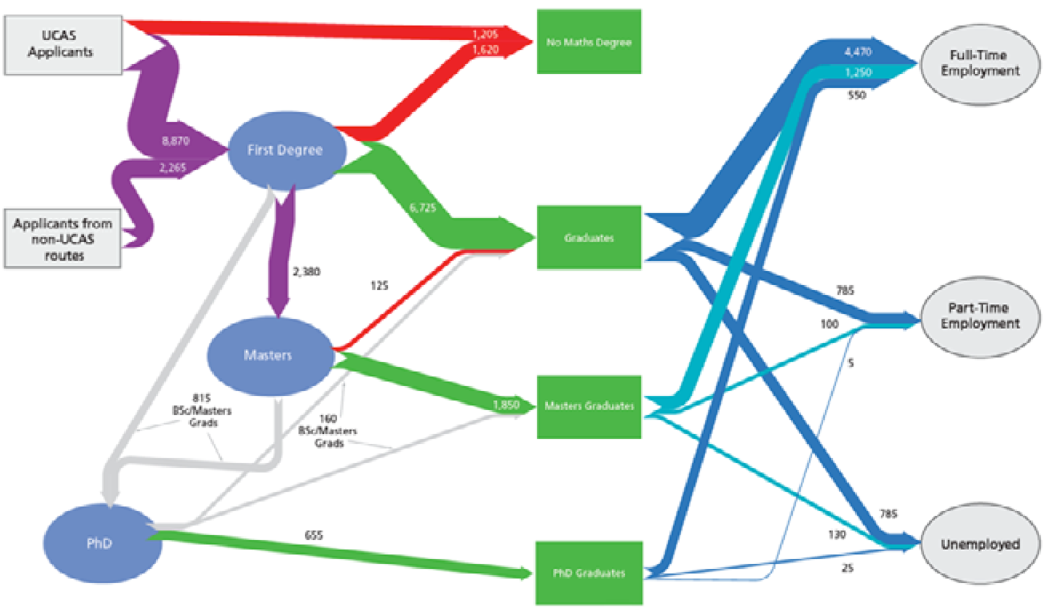
Recent figures from the Department for Education[8] indicate that UK students graduating with MS degrees command the fourth highest median earnings five years after graduation. Interestingly, MS graduates from UK HEIs command the highest median earnings when domiciled overseas. The CMS report in 2015 concluded that, 'There is a need for a healthy pipeline of individuals who are mathematically skilled and trained at all levels to inform a huge section of industry and employment.'

An increase of at least 100 extra PhD places a year in the mathematical sciences would strengthen the supply of the skilled graduates UK industry requires to address 21st century challenges. Broader skill requirements, regularly cited by industry, include coding, problem solving and business and communication skills, as noted by Professor Sir David Spiegelhalter in the area of statistics:

'UK Statistical inference is a major part of mathematical sciences, and also forms a major component of data science, machine learning and artificial intelligence. Statistics is essentially an enabling technology, and it is natural that there is already a wide range of knowledge exchange being carried out.

There is, however, a major shortage of trained people at the deeper end of statistical methodology, and the way to improve knowledge exchange in statistical science is to boost the number of people capable of engaging. This means providing more Centres for Doctoral Training, and more postdoctoral positions with attractive terms and conditions. For example, the current 10 PhD places per year in statistics would need to be at least doubled.'

Figure 3. The mathematical sciences people pipeline (reproduced from CMS, 2015)



[8]See https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/690859/SFR15_2018_Main_text.pdf

Recommendation: Government and universities should create, at a minimum, 100 additional PhD places per year dedicated to training mathematical scientists looking to generate impact with their work. These PhDs should have a greater emphasis on breadth in training, with business and computer coding skills included in addition to deep mathematical expertise.

> ‘Mathematical departments in universities produce people who go out into the world and change the world, both in industry and also by crossing into and transforming other disciplines.’

Knowledge exchange is a ‘people business’ – it is most effective when people are able to interact closely. It is therefore essential that the skills required to network and collaborate effectively are in place, and that career progression is strongly supported for individuals who choose to maximise the impact of their work by engaging in knowledge exchange, cross-cutting and interdisciplinary work, or work for a period of time in industry. Ensuring that knowledge exchange creates powerful economic and social impact requires a strong underpinning support for those engaged in such activities and a number of our recommendations are intended to create an environment in which stakeholder incentives are strongly aligned to achieve this goal.

Professor Sir Adrian Smith recently reviewed mathematics education for 16-18-year-olds in England in view of the ‘increasing importance of mathematical and quantitative skills to the future workforce’ (Smith, 2017). The present review is primarily focused on the importance to mathematical sciences KE of research capacity and the people pipeline at PhD level and above. However, we recognise the enduring importance of a strong supply of well-trained students to universities and would like to reiterate the critical importance of early education as a national strategic priority with impact throughout the economy. Issues surrounding individual academic researchers’ time and career progression with regards to KE are discussed in Section Three.

PhD studentships, student placements and internships, both at undergraduate and postgraduate level, are effective KE routes. Nevertheless, universities differ in the extent to which these routes are used to develop relationships with research users. Such placements and other broadening-skills training activities, such as research ethics, interdisciplinary communication, computer coding, public engagement and project management, are essential in inculcating an aptitude for and interest in KE among mathematical sciences researchers at an early career stage; see also EPSRC’s report on the people pipeline in the mathematical sciences (EPSRC, 2014). A significant majority of mathematics graduates, including those with doctorates, do not go on to careers in academia, so it is vital for the UK economy that their training is the best possible preparation for other careers, for example in business and finance, in addition to deep mathematical expertise. Their skill sets, rather than their research-paper output, are of course the key requirements for industry. A frequent output of a mathematical investigation is an algorithm or code designed to compute a result. Furthermore the UK digital economy is currently growing at twice the rate of the broader economy and requires highly skilled workers for highly paid jobs. Mathematicians often excel intellectually in this environment but are at a disadvantage if unable to code. Coding skills should form a core part of the skill set for all MS students.

> There is an imperative need to train young researchers in KE.

Recommendation: Better provision should be made for early-stage training of mathematicians in KE and problem framing/solving, at undergraduate, masters, PhD and postdoctoral levels. Project/thesis work should increasingly be undertaken in partnership with government or a commercial organisation, with increasing use of internship as a mechanism.

Recommendation: All mathematics students should acquire a working knowledge of at least one programming language.

Free movement of skilled people between academia and industry is important for a thriving KE environment. Creating a supportive environment, which empowers people to leave and return to academia, is essential. This observation was also made in the IRMS (EPSRC, 2011).

> Compared to the United States, for example, there is very little professional mobility between academia and the private sector across all disciplines in the UK. The effect is even more strongly felt in the mathematical sciences as compared ... to engineering or economics.

Time spent in industry can currently be seen as detrimental to academic career progression with its strong emphasis on quantity of academic outputs; this point is discussed further in Section Three. As greater emphasis gets placed on the role of impact in academic output, so the utility of closer engagement with industry increases. A variety of mechanisms, including the greater use of visiting professorships, springboard fellowships, placements and closer engagement with mathematicians in industry, are to be encouraged. In addition, targeted appointments to coordinate and deliver KE activities are not feasible or prioritised in all UK universities, although there have been recent examples at the Universities of Bath, Cambridge, Edinburgh, Manchester and Strathclyde.

Recommendation: Incentives should be created to enable two-way movement of researchers between academia, industry and government.

A great positive of the UK environment is the willingness of UK industry to employ mathematics graduates, in contrast with other nations.

> A major strength supporting knowledge exchange is that UK businesses are very willing to employ mathematics graduates. In fact, mathematics is one of the highest rated degree courses for future employability and subsequent earnings. This contrasts with the position in many competitor countries, where employers often prefer more applied science training. One advantage of this enlightened approach, is that UK companies often have people who know what mathematical sciences research is, know when and how to seek the collaboration of academic mathematicians, and are keen to do so.

Relationships and continuity of contact are critical; for example, mathematics alumni employed in UK industry are often useful first points of contact for academia for building links. Better and more consistent use should be made of these potential links; see also Section 2.4.

Lifelong learning is an increasing feature of modern life, and universities have an excellent opportunity to facilitate alumni in keeping abreast of novel and powerful advances in mathematical technologies, via Massive Open Online Courses (MOOCs) for example. Addressing future issues of knowledge exchange in an era of rapidly expanding mathematical technologies is essential and should increasingly form a part of the knowledge exchange landscape in the UK.

> Keeping strong relationships with university alumni, particularly from MSc and PhD programmes, supports this experience and builds on the strong relationships generally built between students and academic staff once at postgraduate level.

Recommendation: Mechanisms to generate systematic and long-term relationship building and engagement with alumni should be created.

Mathematical Sciences in Government – A Personal Reflection by Professor Sir Bernard Silverman

The influence and importance of mathematics is pervasive, not only within the familiar realms of science and of finance, but also in many other areas of importance to society. My own work as Chief Scientific Adviser to the Home Office in the seven years from 2010 covered all aspects of science, broadly interpreted, but it was interesting for me, and perhaps surprising for others, how much of the work involved mathematics.

My own discipline of statistics has been part of the work of government for centuries and indeed the term 'statistics' was coined for this reason. Government statistical analysis is generally about the accurate and timely production of figures such as economic statistics, the statistics of crime and migration, and the underlying population statistics of the census. All these areas continue to need innovative mathematics, especially as we move into the era of big data, but there are other issues where particular tools have played a special role.

Modern slavery is a worldwide scourge and one which our own government is determined to fight. There are many millions of victims worldwide, but how many of those are in the UK? That was a question I was asked to look at in the run up to the Modern Slavery Act 2015. The National Crime Agency were aware of 2,744 victims in the year 2013, and using a method called Multiple Systems Estimation, by looking closely at the National Crime Agency data, I was able to fit a mathematical model which gave an overall estimate of 10,000 to 13,000 including the 'dark figure' of cases which the Agency did not know about. The method works by generalising an old idea, capture-recapture, and depends on grouping the sources into a number of lists (local authorities, charities/NGOs, government organisations, the police, the general public) and looking at the numbers of cases on each possible overlap of these five lists.

Another area where a careful statistical model was useful was in the Protection of Freedoms Act 2011. This concerned the period for which it can be considered proportionate to hold an individual's DNA profile on the National DNA database, if that individual has been arrested but not charged. The question then arises how likely it is that such a person will be subsequently sanctioned for an offence. A very careful analysis of the police national records showed that for the first three years after their original arrest, there is indeed a higher probability of this than for the overall population. However, after three years have elapsed the risk decays to that of the general population, presumably because by then those who simply got away with it the first time actually get caught! This figure of three years is now enshrined in legislation.

Operational research is another aspect of mathematics that is of great importance in providing efficient public services. Building good mathematical models of things, like the demand for passports or the queues that develop at border crossings, allows for proper allocation of resources, so that customers do not have to wait too long, while time and money are not wasted by having staff being idle. Another area is the modelling of epidemics so that the right decisions can be taken if there is an outbreak of a serious human or animal disease. For example, in 2001 there were very serious policy issues during the foot-and-mouth epidemic about which animals should be culled; decisions had to be made at speed during the progress of the epidemic, but retrospective analysis showed that the rather draconian measures were indeed necessary.

These are just a few examples out of many (some of them classified) where mathematics helps the work of government. Mathematicians themselves will not be surprised, but I hope that these examples give a flavour of the many areas reached by mathematics that are perhaps not well known or obvious at first sight.

2.4 International Comparisons

It has been helpful in this review to consider the KE landscapes for the mathematical sciences in a number of other countries, including the USA, Germany, France, the Netherlands and Ireland. These countries were selected as they have useful points of comparison with the UK research and innovation system. One general point to emerge from their analysis is that it is not just the UK that has recognised the importance of the mathematical sciences to the economy (cf. Deloitte 2012); see, for example:

'The jobs affected by mathematics have a strong added value (15% of GNP and 9% of employment) and are increasing in number (+0.9% per year from 2009 to 2012 vs. +0.5% for overall employment). ... 44% of key technologies, identified as such by government reports, are strongly affected by progress in mathematics' –(AMIES, France, 2015)

'The full-time equivalent of about 900,000 highly educated employees use mathematical sciences in the Netherlands.... [The mathematical sciences] are estimated to create another 1.4 million jobs, resulting in ... up to 26% [of] total employment. Because these are high income jobs, the economic contribution of mathematical sciences is even higher, representing around 30% of Dutch national income.' –(Deloitte, Netherlands, 2014)

'We are convinced that the mathematical and computational sciences have contributed and will continue to contribute to the nation's economy by providing new knowledge and new ways of doing business.' –(Society for Industrial and Applied Mathematics, USA, 2012)

Some countries have a stronger culture of research and development investment directly by industry than is typically the case in the UK, for example, the USA and France, which naturally leads to richer and deeper KE between academia and industry. Another key cultural difference is the extent to which universities maintain relationships with their alumni. Some US universities with strong KE track records, for example Stanford University, invite their alumni to events in which they are offered exposure to the latest advances in the mathematical sciences and data science and which provide contact time for development of new collaborative projects (see Section 2.3).

Germany is a particularly useful comparison country because of the high international standing of both its research and industrial sectors. In Germany there are two other groups, in addition to universities, that play an important role in applied research and KE. The first of these are the Universities of Applied Sciences (UASs; in German *Fachhochschulen*), which focus on teaching professional skills, and where appointment to a professorship is typically only possible for those with a track record in industry. All of their programmes have a strong emphasis on KE. Mathematics groups in UASs tend to be either embedded in engineering departments or focused on applied mathematics, statistics, operational research, etc.

The other key group are the Fraunhofer Institutes, on which the UK's Catapult centres were loosely modelled, and which focus on a particular field of applied research, for example the Kaiserslautern Fraunhofer Institute in Industrial Mathematics, which has an associated centre in Gothenburg, Sweden. It is also worth noting that the Fraunhofer Institute for Applied Photonics is based at the University of Strathclyde. The Fraunhofer Institutes earn about 70% of their income through contracts with industry or the public sector, with the remainder coming from the German federal and regional governments.

There are other noteworthy features in the German system in support of KE. For example, the federal government provides funding to support research and development in SMEs for projects including academic partners. There is also the Steinbeis Foundation, a national charity which supports the formation and operation of spin-out companies; this mechanism works well for high-quality, low-volume products because of its flexible and responsive approach.

In summary, KE routes in Germany are mostly external to the universities performing basic research; nevertheless the system as a whole is effective. Certainly by the measure of international patents applied for Germany greatly outperforms the UK.[9]

As noted earlier, the emphasis on KE and working with SMEs is well aligned with the UK Industrial Strategy. We advocate that universities work increasingly closely with local SMEs. However, in addition to this valuable regional work, a national facility would provide a number of additional benefits. As a KE-centric organisation it would provide better links between academia and industry, create access to SMEs both directly and via the supply chains of larger businesses, and provide a means for academics to spend significant time on industry-facing projects without joining one particular company. Having a single point of contact for mathematical expertise in a number of fields such as optimisation would also facilitate industry interaction.

The centre should operate across the Technology Readiness Level (TRL) chain, working very closely with universities, existing centres and networks, and provide industry links and expertise on commercialisation. An example of the need for such a function has been provided by Professor Sir David Spiegelhalter:

'A gap concerns research into innovations that lead directly to public benefit. While organisations such as the Alan Turing Institute will be doing excellent basic research, the use of routine data to provide services of value to people in their everyday lives is left primarily to the commercial sector producing proprietary products. Research into the principles of transferring mathematical insights into public good tends to fall between funding programmes, although there is some support from philanthropic sources.'

[9] In 2016 Germany applied for 18,135 international (PCT) patents compared to 5,496 from the UK. Source: https://www.statista.com/statistics/256845/ranking-of-the-10-countries-who-filed-the-most-international-patent-applications/ [Accessed 1 November 2017]

Recommendation: A national centre in impactful mathematics for the UK should be created to work with industry and government to drive mathematical research through to commercialisation. This could be based on existing models, such as the Fraunhofer Institute for Industrial Mathematics in Kaiserslautern or the UK Catapult network, suitably modified to provide national-level integration of low-TRL research from universities and to act as a national KE hub.

Networks and centres are vital infrastructures for the mathematical sciences. They provide focal points for industry engagement, contain critical mass of diverse and relevant skills, and allow for sharing of expertise and resource across geographic regions. Networks allow the UK's geographic spread of expertise to be tapped both locally, via local SMEs, and at a national level via inter-network collaboration.

Two international exemplars of MS KE have been highlighted in this report as case studies. The first is the Mathematics Applications Consortium for Science and Industry (MACSI) based at the University of Limerick in Ireland, which focuses on KE in mathematical modelling, and may be compared with, for example, the Oxford Centre for Industrial Mathematics (OCIAM). The MACSI model could potentially have a regional KE focus in the UK. The second example is the European Research Community on Flow, Turbulence and Combustion, ERCOFTAC, whose founding Secretary General was Lord Julian Hunt of Chesterton. The main aims of ERCOFTAC are to promote joint research between academia and industry, to exchange technical and scientific information concerning basic and applied research, and to develop, validate and maintain numerical codes and databases. The UK Fluids Network is aligned with ERCOFTAC.

Mathematics Applications Consortium for Science and Industry - MACSI

MACSI is a network of mathematical modellers and scientific computational analysts based in Ireland. The aim of the network is to foster new collaborative research and training in mathematical modelling.

- MACSI works closely with scientists and industrial companies around the world on interdisciplinary problems, using mathematical expertise to develop insight and provide innovative solutions.
- MACSI enables industrial companies to improve their products and processes through the application of cutting-edge mathematical modelling techniques.
- To date MACSI has worked with over 30 companies. The research outcomes of many of the joint projects have led to increases in productivity and reduction in costs, which ultimately enhance the competitiveness of industrial partners.
- MACSI also aims to increase awareness among students at all levels of the role of mathematics in modern industry.

European Research Community on Flow, Turbulence and Combustion - ERCOFTAC

The main aims of ERCOFTAC are to promote joint efforts of research institutes and industry partners who are active in all aspects of flow, turbulence and combustion research and innovation, with the object of exchanging technical and scientific information concerning basic and applied research, and the development, validation and maintenance of numerical codes and databases. ERCOFTAC promotes industrial application of the research by means of new kinds of collaboration between industry, governments, professional societies and research institutes. ERCOFTAC Pilot Centres in several European countries act as centres for collaboration, simulation and application of research. ERCOFTAC Special Interest Groups support well-coordinated research efforts on specific topics in flow, turbulence and combustion. Within the UK the UK Fluids Network (https://fluids.ac.uk) is the ERCOFTAC UK Pilot Centre, aiming to align the UK and European activities and to centralise the administrative activity. The annual Osborne Reynolds Day Research Student Award is a key event in the ERCOFTAC UK calendar.

Recommendation: Funds should be made available for regional KE centres and/or thematic KE networks following successful models such as the Turing Gateway to Mathematics, the UK fluids network (ERCOFTAC), the University of Bath's IMI and the University of Oxford's OCIAM.

2.5 UK Landscape – Mathematical Sciences Profile and Branding

A national voice for the mathematical sciences in the UK is critically important, for example to provide a unified response to new initiatives such as the Industrial Strategy Challenge Fund, or to promote the importance and successes of mathematical sciences research.

> ‘There are no uniform mechanisms for the mathematical sciences community to provide collective responses to new initiatives.’

Equally it is important to help potential users of mathematical sciences research to engage more effectively with the academic research community. The existing learned societies for mathematics fill important roles in their own niches, but do not collectively provide a recognisable voice after the fashion of the Institute of Physics or the Royal Society of Chemistry, for example.[10] The Council for the Mathematical Sciences (CMS) was established in 2001 'to develop, influence and respond to UK policy issues that affect the mathematical sciences in higher education and research.'[11] As the Royal Academy of Engineering has shown, it is neither necessary nor necessarily desirable to merge existing learned societies, but substantial benefits accrue from having a single Academy which provides a coherent interface between academia and industry, and which interacts with government to generate effective policy. Similarly, we argue that a new Academy for the Mathematical Sciences would create a more coherent framework for the mathematical sciences in the UK, both by working with existing learned societies and also by providing closer links to industry and government.

The existence of the CMS and the valuable work that it does demonstrates that such a body is essential for the effective functioning of the MS community. The critical roles that the new Academy would fulfil are:

1. Driving policy creation and support for the UK MS community in delivering maximal positive societal and economic impact from research.
2. Developing, owning and delivering a national vision for MS, as a point of contact between industry, government and academia at a senior level.
3. Providing a focal point and forum for all areas of the MS community to engage, whilst supporting and nurturing existing societies, not duplicating their activities.

Given the central importance of the Industrial Strategy and the Challenge Funds, such as ISCF and GCRF as well as future opportunities, the Academy should work with groups such as Innovate UK and the Alan Turing Institute to ensure that the mathematical sciences are deeply embedded in national challenges and that the full power of the mathematical sciences is brought to bear on key initiatives. Furthermore, the Academy should seek to work with CEOs and boards of UK companies to help them understand the relevance and application of mathematics in their business or organisation. These people are the decision makers and should be a focus for the Academy's activities. To achieve all these aims the new organisation will require a significant increase in scale and resource compared to that of the CMS, including a dedicated funding stream and staff to support it.

Recommendation: An Academy for the Mathematical Sciences (MS) should be established in order to facilitate links between academia, government and industry. The Academy should act as the focal point and coordinating centre for the community and draw on the deep expertise of the existing learned societies.

As noted in Section 2.1, KE funding for the mathematical sciences in the UK often scales with research funding and is therefore low compared with other STEM disciplines. Substantial benefits would also accrue from deeper MS engagement in high-profile, interdisciplinary research consortia, in particular in leadership roles and in challenge-based funding initiatives.

[10] In 2008 a proposed merger of the London Mathematical Society (LMS) and the Institute of Mathematics and its Applications (IMA) came to nothing following a vote by the membership of both societies, where the proposal was rejected by the LMS membership
[11] From CMS website: http://www.cms.ac.uk/

This requires better integration with groups such as Innovate UK to ensure that the full power of UK mathematical sciences is brought to bear effectively on challenges of national and international importance.

Recommendation: Innovate UK should actively seek to create mechanisms within the Industrial Strategy Challenge Fund (ISCF) and Small Business Research Initiative (SBRI) that encourage industry to engage and form partnerships with MS experts.

Data science is an increasingly important source of mathematical challenges and 21st century mathematics will increasingly focus on issues faced when processing extremely large datasets. Much of the value in data is in finding patterns, recognising things (essential for autonomous cars, for example) and making accurate predictions while assessing the errors of estimates. Vast datasets of this sort range from weather data to traffic data, social media, medical genomics for novel drug discovery and personalised medicine or medical imaging for the early detection of cancers. The mathematical sciences are uniquely positioned to provide new insights, methods, structures and algorithms to tackle these hard challenges in mathematical data science and closer integration of mathematics with the National Centre for Data Science, the Alan Turing Institute (ATI), is to be encouraged. We recommend that mechanisms be put in place to enable far more early-career mathematicians to engage with the ATI. Analogous interactions should be encouraged in other specialist areas.

Recommendation: MS masters and PhD students with an interest in specialist research areas should have the opportunity to engage with national-level initiatives, e.g. researchers in artificial intelligence (AI) with the Alan Turing Institute, in operational research (OR) with government and Dstl, in materials modelling with the Royce Institute, and various subjects with specific Catapult centres.

The prevalent sub-division of mathematical disciplines in UK universities may be helpful for internal university structures but is unhelpful to fostering a common identity and profile for mathematical sciences researchers, i.e. a mathematical sciences 'brand.' 'Branding' issues impact not only the perception and funding of mathematics, but also make it challenging for industry and government to identify mathematics as a key element in many areas of strategic operation. There are a number of reasons for this. In part, impactful mathematics is used in a wide range of other disciplines ranging from engineering and physics through to genomics and weather forecasting, where it is often rebadged and renamed. Much current 'machine learning' and 'econometrics' consists of methodologies directly drawn from statistics, for example. It is extremely valuable that mathematics is used in this way and is a powerful exemplar of KE in mathematical sciences. However, the resultant rebadging of mathematics increasingly makes the mathematical sciences per se 'invisible' to end users who may have little appreciation of the original source of the algorithms or methods that they are drawing on. As a result, potential users of mathematics are often unaware that mathematical methods or the mathematical sciences provide a source of novel solutions to hard problems that might be highly productive for them. This issue needs to be addressed if KE in the UK is to provide greater impact through deeper and broader uptake in both industry and government. The sensitive nature of much work undertaken for national defence and national security also serves to make the use of mathematics 'invisible' despite its pivotal role.

A further issue specific to the mathematical sciences is that government tax relief for science and technology research and development in small businesses is often perceived to be excluded for the mathematical sciences, based on the HMRC's own published definitions of science and technology:[12]

- *'Science is the systematic study of the nature and behaviour of the physical and material universe.' ... 'Mathematical techniques are frequently used in science but mathematical advances in and of themselves are not science unless they are advances in representing the nature and behaviour of the physical and material universe.'*
- *'Technology is the practical application of scientific principles and knowledge, where 'scientific' is based on the definition of science above.'*

> 'Bringing mathematics into line with other sciences in the treatment of tax credits would make negligible difference to the public finances, but would send a powerful signal to business that mathematics is worth investing in. We see no reason why this anomaly should persist and every reason why it should be corrected.'

> 'We need to challenge ... the perception that mathematics is NOT at the heart of industry.'

It is the conclusion of this review that this change in the tax credit system would make the UK a more attractive place for businesses to invest in and carry out research and development, particularly small and medium enterprises (SMEs). This will clearly have attendant benefits for the UK economy and job creation.

Recommendation: The mathematical sciences should be encompassed in the HMRC definition of science and technology and included in the tax-credit scheme.

[12] *http://www.hmrc.gov.uk/gds/cird/attachments/rdsimpleguide.pdf*

The Mathematics of Cities – Professor Sir Alan Wilson, The Alan Turing Institute

Cities are focal points of social and economic life and development. We need to understand cities – that is, build a science of cities – and to use this understanding to be able to articulate present and future challenges and, where appropriate and effective, to plan. Much of this science can be represented mathematically and translated into computer models. These models then become the urban equivalents of flight simulators and can be used by business, public services and city planning. The key elements of this science are: the people and how they live in cities; the urban economy; how cities can be sustainable; urban form; the transport and communications systems; and the governance structure of cities. Mathematical models can be constructed of any one element of the city system; more ambitiously, and with some success, we can build comprehensive models which can then take account of the interdependencies which shape urban development.

There is a long history of city modelling, which has its modern origins in the 1950s with the advent of mainframe computers. The initial developments were in transport and retail, underpinning the cost-benefit analysis of large projects – public or commercial. To fix ideas, consider, in broad terms, the retail system in London. There are roughly 600 wards, which we take as places of residence. We may label a typical one as zone i, and number them i = 1, 2, 3, ... , 600. There are around 200 retail centres, labelled j, j = 1, 2, 3, ... , 200. We can represent the flows from consumers to centres as a 600x200 matrix, $\{S_{ij}\}$, measured in money units. The basic model is then:

$$S_{ij} = A_i e_i P_i W_j \exp(-\beta c_{ij}),\ i = 1\text{-}600,\ j = 1\text{-}200,$$

where e_i is per capita expenditure, P_i= the population, W_j the size (as a measure of pulling power) of a centre, and c_{ij} the cost of getting from i to j. The matrix has 120,000 elements but this is handled by one instruction in the computer program. This model can be elaborated and works well down to the store level. This means that retailers can test the impact of opening a new store at a particular location using the model. Planners can use the model to calculate the impact on the high street of out-of-town centres.

This has become routine. What is still at the research front line is to model the evolution of the city in time – in the retail case, the evolution of the structure of retail centres, $\{W_j\}$. This can be done with a differential equation which looks deceptively simple:

$$dW_j/dt = \varepsilon[D_j - kW_j],$$

where D_j is the total revenue attracted to centre j and kW_j represents the cost of running the centre. The equation is saying that if a centre is profitable, it grows; and vice versa. The mathematical challenge is that each D_j depends on all 200 Wjs and so we have 200 simultaneous differential equations in 200 variables!

The core interaction model works because it is underpinned by Boltzmann-like mathematics – averaging over consumer behaviour rather than claiming to predict at the individual level. Progress is being made with the dynamics – essentially they are Lotka-Volterra equations with 200 'species,' retailers, competing for consumers. It becomes possible to estimate the initial minimum size of a new retail centre for it to succeed; or, with a related model, to predict the onset of gentrification in a residential area. Progress in research is facilitated by increasing supplies of good real-time data and increases in computing power. The latter has supported the building of a comprehensive urban model for the whole of the UK – underpinned (among other things) by a road-transport network that has 3.5M nodes and 8.5M links. When this is fully developed, it will offer an analytics capability for every planning authority in the country, something which is only effectively achieved in London at the moment.

The science of cities, represented in mathematical and computer models, is valuable in applications now; big research challenges remain and progress will continue to add to the toolkit with which we can understand the challenges facing cities and plan for better futures. The mathematics of all this is critical.

Government is a key user of the mathematical sciences; see for example the personal reflection of Sir Bernard Silverman given in this report. In order to improve evidence-based decision-making, more accurately assess risks and enhance productivity in an increasingly data-rich world, it will be necessary for government to ensure that it is making full use of the rich community of mathematical scientists both within government and externally. MS is used in many areas within government, from the Met Office, which employs around 2,000 mathematicians, through to the Office for National Statistics. We recommend that a review of mathematical sciences within government be undertaken with a view to assessing the current state of MS in government service and delivering a government strategy for MS fit for the 21st century.

MS is a potent source of tools for innovating, decision-making and enhancing productivity. Fields such as statistics, operational research, and optimisation can all play a greater role in raising both innovation and productivity across government and in industry.

Recommendation: The Government Chief Scientific Advisor should, in collaboration with the Government Chief Statistician, review the access to, use of, and impact achieved by MS within government.

Recommendation: Deeper links between key government users of MS and academic departments should be encouraged.

Recommendation: Government should actively engage with MS to examine means to utilise MS to improve productivity across the economy.

2.6 Strengthening the UK Landscape

As noted in Section 2.1, funding for mathematical science research in the UK as compared with other STEM disciplines funded in the EPSRC portfolio (engineering and physical sciences) is comparatively low. In 2016-2017, the EPSRC spend on MS research was £19.7m (EPSRC, 2017). However, also as noted earlier, the return on investment expressed as benefit-to-cost ratio is much higher for MS than for other disciplines in the EPSRC portfolio: 588 for mathematical sciences, 246 for chemistry, 88 for engineering, and 31 for physics. The value of this research cannot be overstated, and to use Serre's analogy from Section Two, we must ensure that the shelves remain full so that the UK can compete globally as the knowledge economy we aim to be.

> ‘ [I]t is vital to maintain the flow of novel mathematics: if this dries up, so will [KE]. “Water the roots, or the fruit will wither.” The roots, moreover, are both widespread and deep. ’

> Investment in early-stage, speculative knowledge exchange, allowing for early exploration of ideas is essential, as is the continued funding for fundamental mathematical sciences research.

We are often able to predict that a mathematical breakthrough will be important – but not always. G.H. Hardy, for example, famously boasted in his 'A Mathematician's Apology' of the uselessness of his great love, number theory. Seventy years later, number theory lies at the heart of internet and e-commerce security, fundamental to the functioning of the world economy and of worldwide communications.

> Many of the most important industries 25 years from now will generate value using theories and techniques that have not yet been developed. Long-term UK competitiveness will benefit hugely from being at the forefront of research in the mathematical sciences driven only by the criterion of excellence.

We argue that a tripling of the UKRI research budget for the mathematical sciences would be a relatively modest investment, but would bring the MS funding level up to an equivalent standing to the other EPSRC disciplines (in 2016-17 £82m for physical sciences, £76m for engineering; EPSRC, 2017). It is our strong belief that the potential return on this modest investment would be substantial.

Recommendation: To counter the underfunding of the MS research pipeline and adequately underpin MS in the UK, UK Research and Innovation (UKRI) should look to at least triple the funding going to MS across multiple Research Councils, including but not limited to EPSRC and Innovate UK.

In terms of KE, it is the conclusion of this review that the UK mathematical sciences research community, while highly productive and successful in academic terms (see for example EPSRC, 2017, p.12), is fragmented. There is insufficient infrastructure in place to encourage and empower KE in the widest sense, including engagement between the mathematical sciences sub-disciplines, between the mathematical sciences and other research disciplines, and between academia and industry. The Dowling review found that only 2% of collaborative projects with businesses reported by universities across all research areas were taking place in mathematical sciences departments (Dowling, 2015; Fig.8, p.20). While this 2% figure may involve under-reporting due to the branding issues mentioned above, the Industrial Strategy affords a great opportunity for mathematical sciences to engage in the kind of multidisciplinary work in which mathematicians excel and for which MS can provide powerful insights and solutions. We strongly recommend deeper MS engagement with all stakeholders in the Industrial Strategy, the Industrial Strategy Challenge Fund (ISCF) and the Global Challenges Research Fund (GCRF). A similar point was made in the 2010 International Review of Mathematical Sciences (EPSRC, 2011):

'Major progress on 'grand challenge' problems will almost certainly require substantial involvement, from the beginning, by mathematical sciences researchers. Otherwise, there is a danger that the proposed research may boil down to applying standard mathematical sciences techniques to a new problem rather than seeking a fresh look from the perspective of an expert. Thoughtful advice from mathematical sciences researchers should be sought to ensure that proposed teams for multidisciplinary initiatives represent an appropriate scientific balance.'

If the UK is to maximise the benefits that can accrue by fully leveraging UK mathematical expertise it is essential that infrastructure is put in place which will facilitate knowledge exchange and generate impact at a national and international level. For example, Professor Sir David Spiegelhalter notes:

'Another major opportunity for knowledge exchange is dealing with the challenges of the collecting and analysing data on the indicators monitoring the Sustainable Development Goals.'

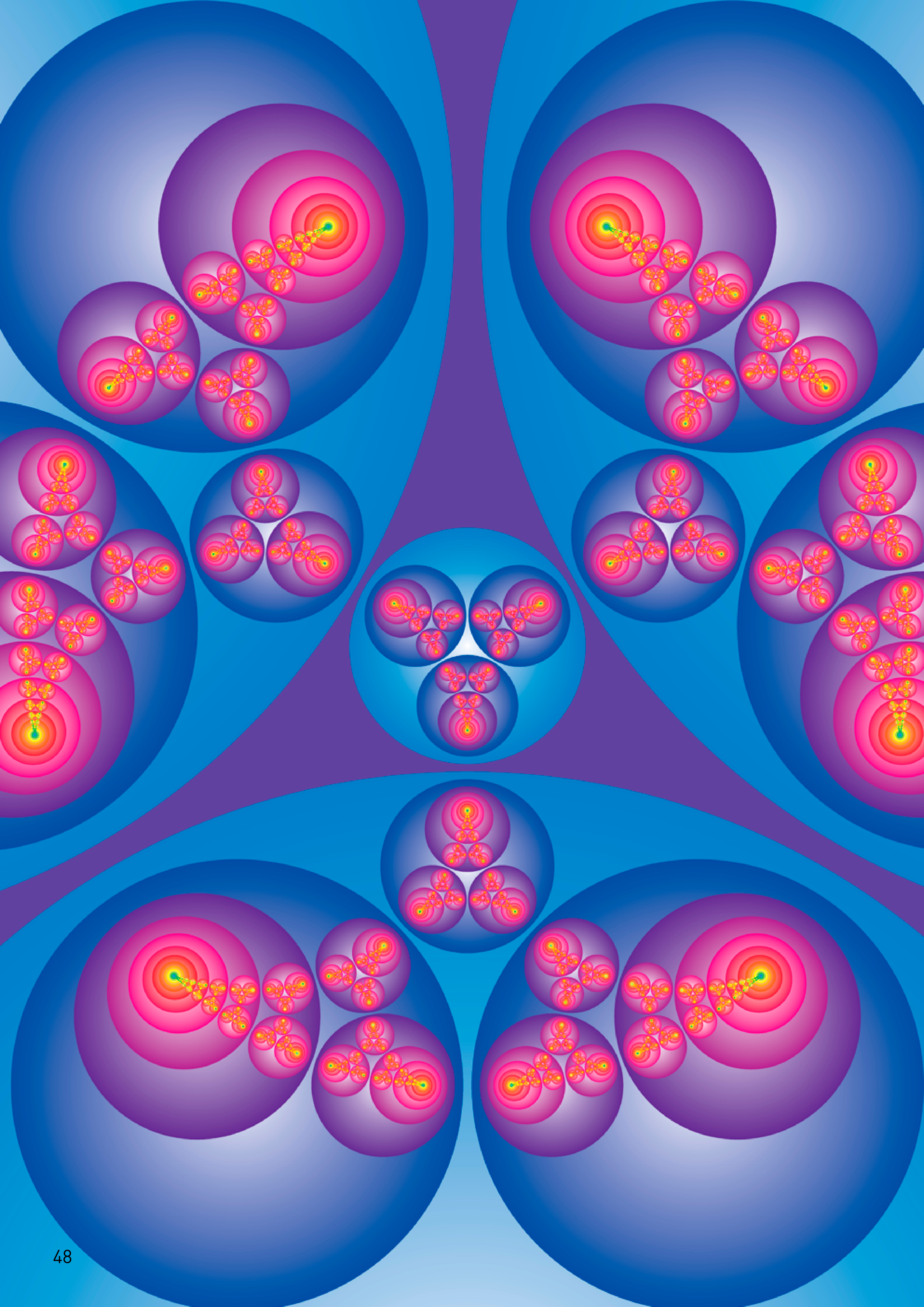
Recommendation: Awareness should be raised within the mathematical sciences community of wider research challenges and societal challenges (including the sustainable development goals addressed by the Global Challenges Research Fund, GCRF) and deeper integration of mathematics should be promoted within industrial challenges (including the Industrial Strategy Challenge Fund, ISCF).

In addition, we believe that mathematics traditionally considered to be 'foundational' can play a far greater role in creating impact than is currently the case. When used in this way 'foundational' mathematics can have profound impact. As introduced in Section One, a key exemplar is the Heilbronn Institute which tackles hard problems critical for national security.

The **Heilbronn Institute for Mathematical Research** has demonstrated that mathematicians focused on sub-disciplines traditionally considering as 'fundamental/pure' mathematics can very effectively engage with important real-world challenges and generate significant impact. Researchers at Heilbronn spend half of their research time undertaking work on behalf of GCHQ with a view to enhancing UK national security, an essential undertaking at a time of increasing terrorist threats. The other half of their time is spent doing personal academic research. Fellowship applications are oversubscribed by a ratio of approximately 20 applications for each post, and Heilbronn researchers have on average maintained the same rate of output of academic papers as academics at a similar career stage based in universities, despite the reduced time available to them (see for example, Heilbronn, 2018). In addition Heilbronn is increasingly seen as a valuable way to move from academia to an 'impact-focused' role and eventually back again should they choose to do so. It therefore serves as an excellent model of engagement for the 'pure/fundamental' community. This community represents approximately 30% of mathematicians, many of the very highest calibre, and engaging them on the important societal challenges facing the UK should be seen as a valuable use of a precious national resource.

The Heilbronn Institute addresses low-TRL, hard problems for a customer who is able to integrate and exploit the solutions at a higher TRL. One or more new centres in different problem domains, not necessarily working on sensitive challenges, but dealing with important and difficult issues, could usefully be created for other customers requiring such research.

Recommendation: There should be at least one national centre, based on the Heilbronn Institute model, to better enable mathematicians focused on fundamental research to engage directly with government and/or industry.



SECTION 3:

Impactful mathematics – what does good knowledge exchange in the mathematical sciences look like?

3.1 Motivations and aspirations

For knowledge exchange to function effectively the motivations and incentives of stakeholders need to be closely aligned. Meagher and Martin (2017) explored in detail the nature of KE in MS from the perspective of HEIs. Through the evidence gathered in this review we are able to add a perspective from industry and public-sector users. Our survey on motivations and aspirations for engaging in KE (see Annex 5) has shown that there is a great deal of KE being undertaken in the mathematical sciences: 44%, 60% and 60% of business, public sector, and academic respondents respectively claim to have engaged in KE in the MS over 10 times in the last five years.

The most common industry motivations for engaging with the mathematical sciences are: seeking new insights, gaining access to expertise not available in their organisation, solving a specific problem, gaining help with horizon scanning/new ideas, and building internal capacity. Small businesses identify motivations including the opportunity to progress their commercial offering, create jobs, increase revenue, and save costs through the insights generated by mathematical sciences collaboration. A further motivation cited by industry is the beneficial publicity generated by successful KE activities. Large companies often indicate more of a focus on idea exploration through learning and new insight: for example, evaluating the success of a KE activity by asking, 'Will the interaction provide us with ideas at the leading edge?'

The academic research community is typically motivated by a wider range of drivers, including altruism, curiosity, increased visibility, problem solving, creativity, generation of academic papers, long-term collaborations, and the opportunity to deliver prestigious conference presentations. Unsurprisingly, perhaps, the most common motivation for academics to engage is having a scientific interest in the problem. The Research Excellence Framework (REF) provides a strong motivation for university researchers to engage in KE in general terms. Public-sector drivers include the wish to deliver societal change, introduce better public services, benefit from evidence-based policy making, access expertise in fields such as risk, and reputational uplift.

As already noted, KE in the mathematical sciences, as in other fields, is largely about people and relationships, and that personal motivation for engagement in KE is critical to a successful outcome. The National Centre for Universities and Business (NCUB), in their 2016 report on Knowledge Exchange in UK Universities, found that KE and 'engagement is a recurrent persistent activity … across a full range of disciplines.' Equally, however, they noted that 'non-engagement also persists' (Lawson *et al.*, 2016).

3.2 Enablers and Incentives

SMEs cite the following key enablers for KE: workshops and events for maintaining contacts, trusted existing links, and placements, such as researchers in residence and student projects. Underpinning these modes of engagement, mutual understanding and transparency of drivers are very important. For SMEs the key to success is seen to be academic researchers' ability to be pragmatic about the choice of approaches used to solve a problem. Helping SMEs to develop and flourish locally is a key goal of the Industrial Strategy.[13] In order for the mathematical sciences to have greater impact in this regard it is necessary to deepen and strengthen links with local SMEs, turn the often ad-hoc nature of engagement into a core value that is professionally managed within universities and provide better support both for SMEs and researchers engaging with them. A hub-and-spoke model with regional KE centres would facilitate engagement and allow SMEs to more easily engage across a national spectrum.

In addition, augmenting the KE capacity within university departments can be achieved in a number of ways. The creation of centres for KE, possibly attached to and closely interacting with Centre for Doctoral Training (CDTs), would enable a more focused approach than is typically possible at present. The University of Bath provides a good example of the embedding of a specialized KE function that works alongside a mathematics CDT and engages across a wide range of academic disciplines within the university as well as SMEs in the Bath region.

We recommend a number of measures to enhance KE into the SME space. Large companies typically have several thousand suppliers within their supply chain. Enhancing the value of supply chains by improved innovation at every stage generates significant added value and we recommend that as part of government initiatives on productivity which are beginning to address supply-chain innovation, the role of mathematical sciences such as operational research, statistics, planning and optimization across the supply chain of major companies should be central.

Study Group Workshops for Industry were initially created in Oxford in 1968. These Study Groups bring together academics and industry in a structured multi-day format where representatives from industry present problems and work with mathematical scientists to brainstorm ideas and work towards practical solutions.[14] The following quotation from the 2010 IRMS originally referred to Industrial Mathematics more generally, but describes well the Study Group format: *'[It is] "multidisciplinary squared", in that it typically includes more than one area of the mathematical sciences as well as more than one area of science and engineering, plus complex constraints (often imprecise and shifting) imposed by the business environment.'*

Study Groups provide a setting for SMEs to engage with a broad spectrum of mathematicians, typically to innovate new products, solve hard problems on which they are currently 'stuck,' or develop a low-cost method for enhancing an existing product or process. This UK model for KE has been successfully exported around the world and is used extensively in China, for example. It allows PhD students to work alongside experienced senior researchers on novel problems and greatly enhances their own problem-solving skills.

Recommendation: Incentives for academic engagement with local SMEs should be created.

Recommendation: Resources for workshops with industry should be broadened and increased. In particular the Mathematical Study Groups with Industry should be expanded in scope.

[13] *https://www.gov.uk/government/publications/industrial-strategy-building-a-britain-fit-for-the-future*

[14] *An overview of the Study Group concept can be found at the website www.maths-in-industry.org maintained at the University of Oxford*

Large businesses cite the need for researchers to have knowledge of business and academic constraints, including awareness of business governance structures and the factors governing the development side of an R&D project. They also highlight the importance of knowing where academic knowledge centres are located, including critical mass EPSRC investments such as CDTs and Programme Grants; it is clear that such investments provide focal points for users to engage with the mathematical sciences. It is therefore important to showcase the wider capability and expertise available in the research community for businesses. In this context businesses have noted that it is important for academic research centres of excellence to reach out and explain what they have to offer. KE professionals embedded in mathematics departments can play a vital role in facilitating this process at the institutional level. Our evidence has highlighted the usefulness of technology translators in mathematics departments, including examples of good practice at, among others, the Universities of Bath, Manchester, Oxford and Strathclyde.

The new National Academy we recommend will provide a means, working with HEIs, existing learned societies and other UK and international organisations, to enable and coordinate this outreach at a national and international level.

Other valuable means to generate collaborations cited in our survey include: case studies; willingness of collaborators to take a risk on a problem; technology translators as ice-breakers; and funds for pump-priming for more open-ended problems.

Recommendation: Universities should have dedicated teams in mathematics departments to act as facilitators and KE translators. These should be connected to central KE functions within universities and coordinated through the National Academy.

Opportunities for interdisciplinary working as enablers for KE are also very important and to be encouraged (see also Meagher and Martin, 2017), for example the potential role of colleagues in other disciplines as translators of mathematical knowledge and ideas.

Recommendation: Strong incentives should be put in place for cross-disciplinary work between the mathematical sciences and other disciplines.

In general terms, contact time is a critical feature of success for KE, and therefore opportunities for face-to-face discussion are paramount. This point is explored further below.

3.3 Creating Incentives for and Removing Barriers to Impactful KE

Academics have many calls on their time. It is essential that adequate time, recognition and resources be available to ensure that they are incentivised to focus on KE as part of their career, and that the choice to work toward impact is recognised in career progression. A critical factor is often the time needed for effective KE, and there must be benefit, not opportunity cost, to academics as individuals of engaging in these activities. KE must be adequately recognised by their institutions.

Changes to the UK academic landscape in recent years can powerfully support enhanced KE across numerous disciplines and particularly that of impactful mathematics. Two factors are particularly significant: the Stern Review (2016) recognised the value of impactful academic work, reflected in subsequent changes to the REF; and the creation of UKRI after the Nurse Review (2015) provides an opportunity to create a rich UK environment for cross-cutting, interdisciplinary and impactful work. UKRI can support and enhance KE both in the mathematical sciences and in other disciplines by creating an environment in which academics have the right career incentives and structures in place to allow them to build successful careers while focused on impactful work. The importance of building relationships and personal networks was highlighted in the Meagher and Martin study (2017) using a survey of Heads of Department (HoD): *'[O]ver two-thirds (68%) of HoD respondents selected "informal relationships, informal knowledge exchange" as effective in helping to generate impacts, and when asked specifically about relationships, almost all (91%, with half of these strongly agreeing) believe that, in their departments, "those academics who have developed lasting relationships with individual stakeholders have generated the most impacts."'*

> I have found that my knowledge exchange activities have depended strongly on my personal contacts. These take time to build, and the time taken to do this is not generally recognised.

> A researcher might have to consider what effect devoting time to work on "knowledge exchange problems" has on their publication rate and quality, how this would affect their standing for REF submission and how it would affect their career prospects in general. Considerations like this may represent a considerable disincentive for many colleagues.

> No provision is usually made to allow individuals the time to become more engaged in KE activity; it is just expected they do it on top of everything else.

At present, many researchers still consider that promotion and advancement is based on papers published in high-impact-factor academic journals and teaching activities. Publishing case-based impact papers is often perceived as being of lower priority and lower kudos than research papers. Yet many academics wish to spend more time on impactful, cross-disciplinary work. Clearly cross-cutting research both within academia and government should be further stimulated and routes to access simplified and streamlined. This issue is likely to be particularly acute for early-stage career researchers. If the potential for the mathematical sciences in generating impact is to be fully realised, a structural change is required, in line with the intention of the Stern Review. To this end we make the following recommendation.

Recommendation: KE activities should be fully integrated into MS academic careers and career progression. This should include consideration of KE in academic appointment and promotion criteria, as well as mechanisms to incentivize and support KE activities. Mechanisms should include KE accolades and buy-out of teaching time for academics who complete an industry placement to ensure that academic research productivity is maintained.

'[T]he creation of Centres for Doctoral Training has produced centres of critical mass, which have provided important focal points for business to access pools of young researchers. In a subject such as mathematics, where large research teams are less common than in other disciplines, it is difficult for businesses to navigate the research landscape. CDTs help by providing some easily visible landmarks.'

As noted above, if the UK is to maintain excellence and a world-class capability in the mathematical sciences, along with an increased emphasis on impact, it will be necessary to teach a wider range of skills to students and early-stage researchers. CDTs and other academic centres of excellence should seek to embed good practice in KE more closely and engage strongly with local businesses.

Recommendation: PhD training centres and other centres of excellence should integrate knowledge exchange more tightly and seek to interact more extensively with local SMEs and larger businesses.

Within academia we have found that there is a concern regarding lack of funding available in the UK for proof-of-concept studies, in particular for SMEs and public-sector bodies (see Table 2). Awareness of support mechanisms is of course an important factor in their effectiveness. In this respect a national Academy for the Mathematical Sciences would play a key role in raising awareness, providing coordination and in addressing a number of outstanding issues.

University culture around innovation currently acts as a barrier to effective KE in a number of ways:

- Lack of understanding of business drivers and culture;
- Unrealistic university technology transfer offices in terms of KE outcomes;

- Challenges surrounding ownership of intellectual property (IP), for example the potential demotivating effect to academics of universities owning IP. An undue emphasis on IP may be detrimental to KE in the mathematical sciences;
- Complexity/inflexibility of contractual arrangement, for example collaboration agreements.

From Table 2 we observe that common issues for academics, businesses (both large and small), and public organisations relate to lack of time and/or internal resources and the bureaucracy/ inflexibility of their own organisations. Respondents have noted that the inability to progress exciting ideas due to the inflexibility of universities or contractual issues have at times meant that a project has not taken place at all.

Intellectual property (IP) and confidentiality concerns are particularly an issue for large private-sector organisations. These views reinforce the recommendation of the Dowling review that further work is required to optimize universities' approaches to intellectual property and that university technology transfer offices should prioritize KE in general rather than targeting short-term revenue generation (Dowling, 2015; recommendation 19, p.6).

Table 2. Barriers to successful KE activities[15]

Barrier to successful KE activity	Academia	Public sector	Small business (←250 employees)	Large business (→250 employees)
Lack of personal time and/ or internal resources	68	55	54	66
Inability of colleagues to progress collaboration	24	20	44	33
Bureaucracy/inflexibility of own organisation	43	40	46	50
Reaching agreement on IP / confidentiality	25	15	32	48
Unable to meet costs	21	25	34	26
Difficulties in communication	21	10	5	12
None	12	10	20	14

3.4 KE Scales and Timescales

Substantial impact and value is created by ensuring longer-term KE and maintaining productive relationships. Current support mechanisms are typically focused on short-term problem-solving with immediate impacts, whereas early exploration of ideas can be very valuable. Resourcing of projects at short notice is also typically difficult for university groups, for example, redirection of postdoctoral research effort. This effect is likely to be more pronounced in mathematical sciences where funded projects are typically carried out by smaller groups than in other STEM disciplines.

[15] *In each column we give the percentage of respondents by stakeholder group citing each category of barrier*

> Opportunities for pump-priming knowledge exchange activities with other academic disciplines, or schemes that allow a group of researchers to hire a postdoc for six months or so, to explore and investigate a fundamental idea would enable wider and more productive knowledge exchange.

Engagement with industry typically requires greater flexibility and time responsiveness than is usual in academia. Increasing the role of CDTs and creating specialist KE units will provide better mechanisms for both short term and longer engagement. Additionally we recommend greater flexibility for post-doctoral engagement.

3.5 Initiating Activities

Broadening and deepening interactions between academia, industry and government will lead to greater KE impact in the UK. Improving the initiation and maintenance of valuable interactions plays a key role in achieving this ambition.

The review survey has highlighted a broad range of mechanisms for initiating partnerships or activities and there is no single prevailing approach. However, use of existing contacts, either one's own or a colleague's, is the most common method. When asked 'How do you find the industrial/academic collaborators you engage with?' 81% of respondents in large companies selected 'via an existing contact' from a list of options. The selection of this route was similarly high for SMEs (79%), public sector organisations (86%) and academics (84%). Direct approaches, following on from literature or online searches, and networking events also appear to be effective in initiating activities. However, robust mechanisms for identifying appropriate expertise are lacking and this might be particularly difficult for SMEs. A National Centre for KE or hub/spoke model for a future KE network would allow a more consistent mechanism for engagement. Tools such as Konfer,[16] developed by NCUB with HEFCE and the Research Councils, might also prove helpful in the future.

> It can be hard for individual academics to know where to start (do they have the right expertise, how do they find contacts?) and there is no well recognised route to industrial collaboration.

University departments do not always have clear processes for managing direct approaches from research users; this represents a risk that users approaching mathematical sciences researchers may experience variable 'customer service' depending upon the individual

[16] *https://konfer.online/about*

Mathematics in Plant Science and Food Security – Professor Dame Ottoline Leyser, Director, Sainsbury Laboratory Cambridge

Photosynthetic plants and microbes are the foundation for virtually every ecosystem and every agricultural system on the planet. The conversion of light energy to chemical energy by photosynthesis provides both the fuel and raw materials for life on earth. As human populations have grown, so has the demand on both ecosystems and agricultural systems to support human health and wellbeing, such that now achieving environmentally sustainable food security is a central challenge for humanity. Furthermore, the output of historical photosynthesis in the form of fossil fuel provides much of the fuel and raw materials for our industrial activity and the switch to contemporary photosynthesis is an important part of the solution for reducing dependence on fossil fuel, extending further the demands on agriculture. These considerations make it clear that understanding plants is a prerequisite for delivering a sustainable future with a secure supply of high-quality food for all.

Fortunately, as for biology as a whole, the rate of progress in plant science has been dramatically accelerated over recent decades by transformative technological advances, particularly in genomics and diverse live-imaging approaches at scales ranging from remote sensing by satellites to super-resolution microscopy of molecules in cells. While mathematics has always been an important tool for biology, the advent of these technologies has made it completely indispensable.

Firstly, simply capturing the data effectively, reliably and efficiently requires mathematics. For example, identification of particular features in biological images, such as cell boundaries in three-dimensional microscopy images, or aligning images captured over a time series to track features in a growing tissue, can only be efficiently achieved using algorithmic approaches.

Secondly, biological insights can be generated through the identification of correlations in the complex multifactorial datasets generated using these technologies. For example, the explosion in the availability of complete genome sequences driven by advances in sequencing technology has allowed a range of sophisticated statistical analyses linking genetic variation, either within or between species, to phenotypic variation. This has led to the identification of genes underpinning important plant traits, and perhaps more importantly, insights about how evolution has tuned these traits to allow adaptation to new environments.

Thirdly, in combination with well-established approaches, these new tools have provided a wealth of understanding about the molecular players that underpin diverse aspects of plant biology. As a result, the current focus in plant science is shifting from the descriptive analyses of these component parts and their immediate modes of action, to understanding how they interact to deliver the diverse multi-scale properties characteristic of plant life.

Plants function through an astonishing array of systems that constantly monitor and integrate information about the prevailing environment to tune growth, development, defence, metabolism and the associated resource-allocation decisions. Examples range from the timing of key developmental transitions, such as seed germination or flowering, to align with a combination of environmental cues; to adjusting the rate of usage of carbon reserves through the night to ensure adequate supply until dawn, whatever the time of year and however much carbon photosynthesis has captured during the day.

These dynamical systems are characterized by extensive feedback, feed-forward and non-linearity across multiple biological scales. They are impossible to understand without mathematical modelling. Indeed it has rapidly become impossible to make even simple predictions about how these systems will behave in response to perturbation, and thus to test our current understanding of how they work, without formalized modelling.

It is therefore not surprising that progress in plant biology is increasingly dependent on attracting mathematicians, computer scientists, physicists and systems engineers. The ultimate goal is to create truly predictive models of plants growing in agricultural and natural environments that can inform practice in plant breeding, agronomy and conservation. Such models are urgently needed to meet the challenges of the 21st century.

approached. University technology transfer offices (TTOs) are not typically the primary source of KE contacts in the mathematical sciences. The responses to our survey suggest that approaches via knowledge transfer and technology transfer offices are low. When asked 'How do you find the industrial/academic collaborators you engage with?' just 26% of SMEs and large business respondents cited 'via a University knowledge/technology transfer office.' The corresponding figures were slightly higher number for public-sector organisations (32%), whilst only 22% of academic respondents cited this option. This relatively low use of university TTO functions echoes the findings of Meagher and Martin (2017), who note, *'[Knowledge intermediaries] have a low profile in mathematics. Just a 10th (9%) of HoD respondents selected the role from a list of mechanisms [...] Only just over a quarter (27%) of respondents thought they played a useful role in helping to generate impacts and over a third (36%) thought they did not, with the rest neutral.'*

When asked 'Do you feel that you are able to access academic support / different academic disciplines as quickly as needed?' 75% of our academic respondents answered yes, falling to 67% for public-sector organisations; just 58% of businesses respondents said they were able to access support quickly enough.

The recommendations of this review are designed in part to address these issues: creating specialist KE centres immediately creates a central point of contact within a given faculty and ensures a more uniform engagement with industrial partners. A National Centre for KE would have a broader picture of UK-wide expertise and could engage at a more significant level with larger businesses.

It can be hard to track the results and influence of academia-industry collaborations. More generally the processes and tools for tracking mathematical research to the point of impact are not in place, so that recognition for the role of the mathematical sciences in the process is often lost, leading to the REF underestimation cited earlier (Dowling, 2015). The exceptional breadth of mathematical impact in widely differing domains exacerbates these effects.

Recommendation: A more systematic and coordinated approach needs to be adopted to make new and maintain existing KE contacts and to track the outcomes and impacts of KE activities.

3.6 Knowledge Exchange Outputs and Outcomes: Measuring Success

The measurement of success in KE generally is of current concern both to government and to academia. SMEs typically identify beneficial changes in business practices and financial returns from innovations as key measures of success. Larger companies require the assimilation of knowledge via change of internal practices to become 'business as usual.' In other words, impact needs to be driven into the business itself. The ability to monetize and implement ideas is crucial. Key underpinning technologies for delivering change include the delivery of 'know-how' and software that implements mathematical technologies in a useable way. Mathematicians intending to generate impact are well served by acquiring skills in communicating 'know-how' and converting mathematical insights into algorithms and eventually software in prototype form that can be implemented professionally by skilled engineers. As noted above, this review recommends that students acquire skills that will better enable them to fulfil these key roles in KE. Large companies include publications, generation of intellectual property such as patents, solutions to difficult and challenging problems, and insights into novel approaches as having substantial value. Both industry and researchers value 'repeat business' and the establishment of long term relationships. Ensuring adequate time for KE, funding up the TRL chain, and tax credits to stimulate R&D via mathematics, will substantially benefit KE throughout the UK economy.

Value creation is not always measurable in terms of profit. Mathematics has provided underpinning technologies for numerous Olympics medals, enhances our national security, improves our defence capability, enables industrial productivity gains through optimisation technologies, enhances the evidence-based decision-making essential in modern government, and is increasingly important in the medical sciences which underpin the NHS. Recognising these gains and their source in the mathematical sciences is important, and we welcome future work that will improve and enhance the measurement of impact generated by the mathematical sciences. As stated in the Stern Review, 'the new impact element of the REF has contributed to an evolving culture of wider engagement, thereby enhancing delivery of the benefits arising from research.' We believe that implementation of the recommendations of the present review will allow the mathematical sciences to generate substantial and sustained impact, to the benefit of all. The tangible outcomes of the present review should, in line with the spirit of the Stern Review, be audited in five to ten years' time.

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Annexes

1. List of abbreviations
2. Expert Review Committee membership
3. Review Board membership and terms of reference
4. Call for evidence questions and respondents
5. Details of review surveys and list of case studies (published on the EPSRC website)
6. Summary of review community engagement workshops (published on the EPSRC website)

Annex 1: List of abbreviations

AI	Artificial intelligence
CDT	Centre for Doctoral Training
CMS	Council for Mathematical Sciences
Dstl	Defence Science and Technology Laboratory
EPSRC	Engineering and Physical Sciences Research Council
ERCOFTAC	European Research Community on Flow, Turbulence and Combustion
GCHQ	Government Communications Headquarters
GCRF	Global Challenges Research Fund
GDP	Gross domestic product
GVA	Gross value added
HEFCE	Higher Education Funding Council for England
HEI	Higher education institution
HEIF	Higher Education Innovation Funding
HMRC	Her Majesty's Revenue and Customs
IAA	Impact Acceleration Account
ICMS	International Centre for Mathematical Sciences
IMI	Institute for Mathematical Innovation (University of Bath)
INI	Isaac Newton Institute for Mathematical Sciences
IOP	Institute of Physics

IP	Intellectual property
IRMS	International Review of Mathematical Sciences (2010)
ISCF	Industrial Strategy Challenge Fund
KE	Knowledge exchange
KTN	The Knowledge Transfer Network Ltd
MACSI	Mathematics Applications Consortium for Science and Industry (University of Limerick, Republic of Ireland)
MOOC	Massive open online course
MS	Mathematical sciences
NGO	Non-governmental organisation
OCIAM	Oxford Centre for Industrial and Applied Mathematics (University of Oxford)
OECD	Organisation for Economic Cooperation and Development
OR	Operational or operations research
PDE	Partial differential equation
REF	Research Excellence Framework
SBRI	Small Business Research Initiative
SME	Small- and medium-sized enterprise
STFC	Science and Technology Facilities Council
TGM	Turing Gateway to Mathematics (University of Cambridge)
TRL	Technology readiness level
TTO	Technology transfer office
UKRI	UK Research and Innovation

Annex 2. Expert Review Committee Membership

Professor David Abrahams	Isaac Newton Institute
Professor Philip Aston	University of Surrey
Dr Colin Bleak	University of St Andrews
Professor Alan Champneys	University of Bristol
Mr Joseph Connor	Experto Crede Ltd
Professor Rama Cont	Imperial College London
Dr Stephen Corson	University of Strathclyde
Professor Joerg Fliege	University of Southampton
Professor Paul Harper	Cardiff University
Professor Andrew Hogg	University of Bristol
Dr Joanna Jordan	University of Bath
Professor Mark Kelmanson	University of Leeds
Professor Dick Lacey	Home Office
Ms Jane Leeks	Turing Gateway to Mathematics
Professor David Leslie	Lancaster University
Professor Daniel Lesnic	University of Leeds
Professor Bill Lionheart	The University of Manchester
Mr James Lofthouse	Home Office
Professor Gabriel Lord	Heriot-Watt University
Mr Adrian Mardell	Jaguar Land Rover
Professor Jeremy Oakley	University of Sheffield
Dr Richard Pinch	Civil Service
Dr Matthew Revie	University of Strathclyde
Mr Edward Rochead	Ministry of Defence
Dr Sanjiv Sharma	Airbus
Dr Manuchehr Soleimani	University of Bath
Dr Brendan Spillane	University of Warwick
Dr David Standingford	Zenotech Ltd

Annex 3. Review Board Membership and Terms of Reference

Membership

Professor Philip Bond BSc, DEA, FIMA, FInstP (Chair)

The Lord Stern of Brentford, Kt FRS FBA (Co-Chair)

Professor David Abrahams

Professor Dame Glynis Breakwell DBE, DL, FRSA, FAcSS

Professor Martin Bridson FRS

Dr Claire Craig CBE

Professor Jon Keating FRS

Professor Ursula Martin CBE, FREng

Dame Julia Slingo DBE, FRS

Mr Paul Stein

Terms of Reference

1. The role of the Board is to review the draft report, provide expert input into recommendation areas initially suggested, and ultimately provide final approval of the review report for delivery in spring 2018.
2. Appointments to the Board will be for a limited period and will consist of one Board meeting on the 6th December 2017, some preparatory work reviewing the draft report prior to the Board meeting, and post-Board in reviewing the updated, finalised review. A commitment of 1-3 days is therefore envisaged.
3. For the Board meeting, travel, accommodation, and dinner the previous night will be covered. Remuneration is available for work outside of this one day at the EPSRC honorarium rate £160 per day for a maximum of two days per Board member.
4. For those Board members unable to attend the Board meeting, comments on the review draft and recommendations would be appreciated prior to the 6th December 2017.

Annex 4. Call for Evidence Questions and Respondents

Call for evidence questions

1. What are the strengths and weaknesses of the current support mechanisms in place in the UK to assist in knowledge exchange in the mathematical sciences?
2. How is mathematical knowledge exchange currently incentivised and are there any more effective ways to incentivise knowledge exchange?
3. Are there any challenges to knowledge exchange which are unique to the mathematical sciences? What are they? Please also identify opportunities to learn from approaches to knowledge exchange in other disciplines.
4. What are the key opportunities for the UK economy, society, people and knowledge associated with effective knowledge exchange in the mathematical sciences? What do you think the missed opportunities could be? When answering the above, please consider:

a. What are the current opportunities?

b. What are the opportunities looking to the future?

5. Do you have any other comments you wish to feed into the review?

Respondents

Heilbronn Institute for Mathematical Research

Imperial College London

Institute for Mathematics and its Applications

International Centre for Mathematical Sciences

London Mathematical Society

Operational Research Society

Royal Statistical Society

Smith Institute for Industrial Mathematics and Systems Engineering

The University of Manchester

University College London

University of Bath:
(1) Mathematical Sciences
(2) SAMBa Centre for Doctoral Training

University of Edinburgh, School of Mathematics

University of Glasgow, School of Mathematics & Statistics

University of Bristol

University of Leeds, School of Mathematics

University of Leicester

University of Liverpool, Institute for Risk and Uncertainty

University of Nottingham

University of Oxford:
- (1) Mathematical Institute
- (2) Professor Peter Grindrod CBE

University of Sheffield, School of Mathematics & Statistics

University of Sussex, Department of Mathematics

University of St Andrews, School of Mathematics & Statistics

University of Warwick:
- (1) Warwick Centre for Complexity Science
- (2) CDT in Mathematics for Real World Systems
- (3) Professor Robert Mackay

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